

Shell sort algorithm:

1. Sort elements separated by equal intervals in ascending order. For example, put elements 0, 5, 10, ... in ascending order.
2. Decrease the interval and repeat step 1 until the interval size is 1.

Shell Sort Property: Subsequent sorts using a smaller interval do not invalidate sorts performed with a larger interval.

Original	81	94	11	96	12	35	17	95	28	58	41	75	15
After 5-sort	35	17	11	28	12	41	75	15	96	58	81	94	95
After 3-sort	28	17	11	35	12	41	58	15	96	75	81	94	96
After 1-sort	11	12	15	17	28	35	41	58	75	81	94	95	96

Analysis:

1. Choose N to be a power of 2, $N = 2^k$
2. Make the first interval $\frac{N}{2}$
3. Make subsequent intervals one-half the previous interval.
4. In the first interval there are entries in positions 0 and $\frac{N}{2}$. One shift is performed if the entry in position 0 is larger than the entry in position $\frac{N}{2}$.
5. In the second interval there are entries in positions $0, \frac{1}{4}N, \frac{1}{2}N$, and $\frac{3}{4}N$. Please note that the values in positions 0 and $N/2$ are already sorted relative to each other. The maximum number of positions that the element in position $N/4$ can be moved is one. The maximum number of positions that the element in position $3N/4$ can be moved is three. The total number of positions shifted is four
6. In the third interval there are entries in positions $0, N/8, N/4, 3N/8, N/2, 5N/8, 3N/4$, and $7N/8$. The entries in positions $N/8, 3N/8, 5N/8$, and $7N/8$ are unsorted: they can be shifted 1, 3, 5, and 7 positions in the worst case. The total number of positions shifted is 16.
7. A pattern is emerging. The pattern is 1, 4, 16, ...
8. In general, for interval i , there are $\frac{N}{2^i}$ entries to sort. $0 \leq i < k$
9. For the i th interval, there are 2^{2i} positions shifted in the worst case.
10. The total of all positions shifted is $\sum_{i=0}^{k-1} 2^{2i} = \frac{2^{2k} - 1}{3}$
11. Now $k = \log_2 N$
12. Substituting $\log_2 N$ for k into the result of step 10 yields $\frac{2^{2\log_2 N} - 1}{3} = \frac{2^{\log_2 N^2} - 1}{3} = \frac{N^2 - 1}{3}$
13. Shell sort is $O(N^2)$ in the worst case.