

Insertion sort algorithm

1. Assume elements zero (0) through $p-1$ are sorted (in ascending order).
2. Insert the element in position p into its proper place in the preceding (sorted) elements. Now $p+1$ elements are sorted.
3. Perform steps 1 and 2 above starting with element 0 being the entire sorted array from 0 to $p-1$. Element 1 is in position p . Increment p after each iteration of steps 1 and 2 until all the elements in the array have been sorted.

Original	34	8	64	51	32	21	Positions Moved
After $p=1$	8	34	64	51	32	21	1
After $p=2$	8	34	64	51	32	21	0
After $p=3$	8	34	51	64	32	21	1
After $p=4$	8	32	34	51	64	21	3
After $p=5$	8	21	32	34	51	64	4

Table 1. Insertion sort after each pass

Worst case analysis:

1. When $p=1$, the element in position p could be shifted to position 0, requiring one shift.
2. When $p=2$, the element in position p could be shifted to position 0, requiring two shifts.
3. When $p=3$, the element in position p could be shifted to position 0, requiring three shifts.
4. When $p=N-1$, the element in position p could be shifted to position 0, requiring p shifts.

Sum all shifts.

$$\sum_{p=1}^{N-1} p = \frac{N(N-1)}{2} = \frac{1}{2}N^2 - \frac{1}{2}N = O(N^2)$$