

Insertion sort algorithm

1. Assume elements zero (0) through  $p-1$  are sorted (in ascending order).
2. Insert the element in position  $p$  into its proper place in the preceding (sorted) elements. Now  $p+1$  elements are sorted.
3. Perform steps 1 and 2 above starting with element 0 being the entire sorted array from 0 to  $p-1$ . Element 1 is in position  $p$ . Increment  $p$  after each iteration of steps 1 and 2 until all the elements in the array have been sorted.

Original	34	8	64	51	32	21	Positions Moved
After p = 1	8	34	64	51	32	21	1
After p = 2	8	34	64	51	32	21	0
After p = 3	8	34	51	64	32	21	1
After p = 4	8	32	34	51	64	21	3
After p = 5	8	21	32	34	51	64	4

**Table 1.** Insertion sort after each pass

Worst case analysis:

1. When  $p=1$ , the element in position  $p$  could be shifted to position 0, requiring one shift.
2. When  $p=2$ , the element in position  $p$  could be shifted to position 0, requiring two shifts.
3. When  $p=3$ , the element in position  $p$  could be shifted to position 0, requiring three shifts.
4. When  $p=N-1$ , the element in position  $p$  could be shifted to position 0, requiring  $p$  shifts.

Sum all shifts.

$$\sum_{p=1}^{N-1} p = \frac{N(N-1)}{2} = \frac{1}{2}N^2 - \frac{1}{2}N = O(N^2)$$