

1. A graph $G = (V, E)$ consists of a set of vertices, V , and a set of edges, E .
2. Each edge is a pair (v, w) where $v, w \in V$.
3. Edges are sometimes referred to as arcs.
4. Directed edges are represented as ordered pairs (v, w) . The edge is directed from vertex v to vertex w . Vertex v is the origin and vertex w is the destination.

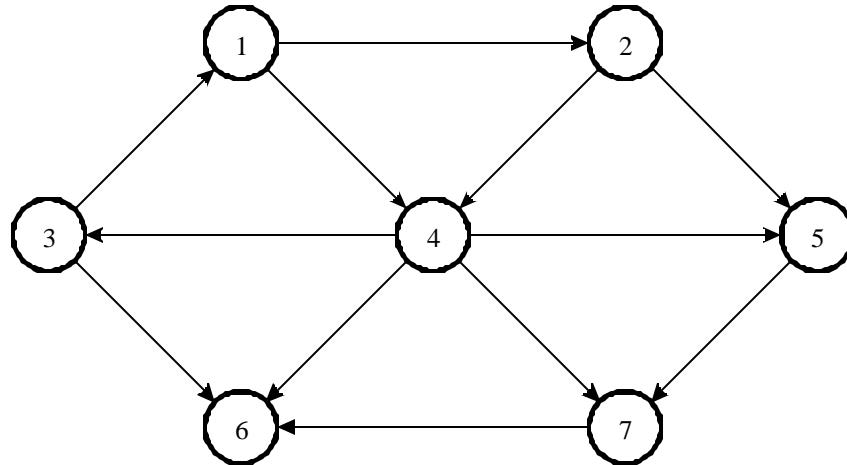


Figure 1. Graph G.

$$V = \{1, 2, 3, 4, 5, 6, 7\}$$

$$E = \{(1,2), (1,4), (2,4), (2,5), (3,1), (3,6), (4,3), (4,5), (4,6), (4,7), (5,7), (7,6)\}$$

		Destination						
		1	2	3	4	5	6	7
Origin	1							
	2							
3								
4								
5								
6								
7								

Matrix representation of edges:

Figure 2. Matrix representation of edges.

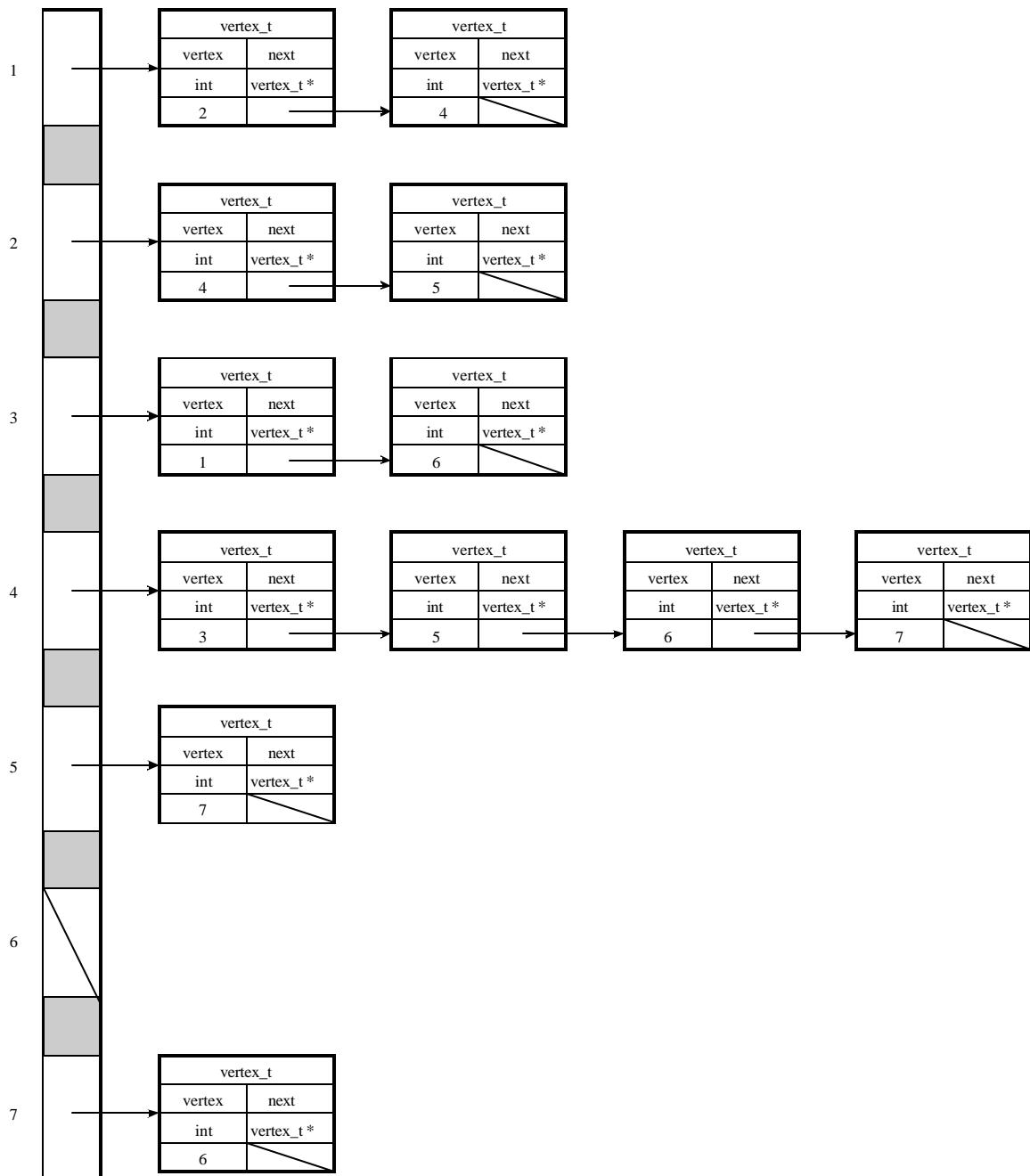


Figure 3. List representation of edges.

Topological Sort

A *topological sort* is an ordering of vertices in a directed acyclic graph, such that if there is a path from v_i to v_j , then v_j appears *after* v_i in the ordering. In the graph in Figure 1, 1, 2, 5, 4, 3, 7, 6 and 1, 2, 5, 4, 7, 3, 6 are both topological orderings.

To perform a topological sort we must define the *indegree* of a vertex. The *indegree* of a vertex v is the number of edges (u,v) . The *indegree* of a vertex v is the number edges directed to v .

Algorithm:

1. Create queue Q having an entry for each vertex in the graph.
2. Enqueue all vertexes having an indegree of zero. Enqueue all vertexes that have only outgoing edges.
3. Loop while queue Q is not empty.
 - 3.1. Dequeue vertex V from queue Q
 - 3.2. For each vertex W adjacent to V
 - 3.2.1. Decrement the indegree of vertex W
 - 3.2.2. Enqueue vertex W if its *indegree* is zero

Vertex	Indegree Before Dequeue #						
	1	2	3	4	5	6	7
1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0
3	2	1	1	1	0	0	0
4	3	2	1	0	0	0	0
5	1	1	0	0	0	0	0
6	3	3	3	3	2	1	0
7	2	2	1	1	0	0	0
Q	{1}	{2}	{5}	{4}	{3,7}	{7}	{6}
Dequeue	1	2	5	4	3	7	6
Adjacent	2,3,4	4,5	4,7	3,6,7	6	6	

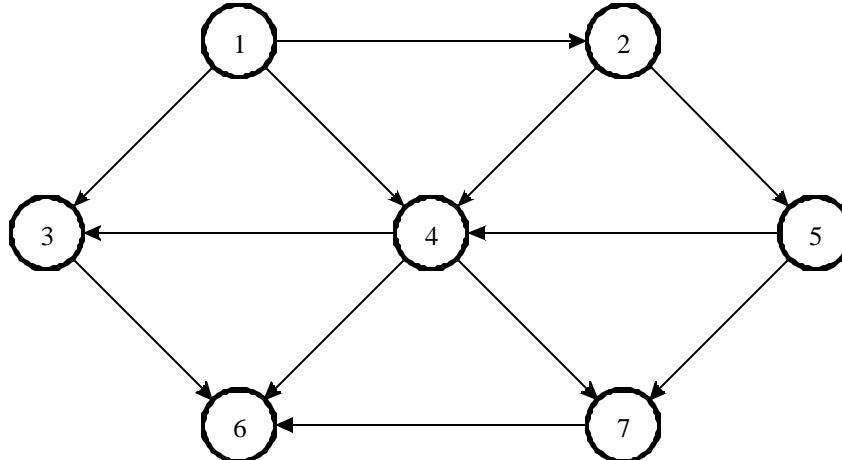


Figure 4. Graph for Topological Sort