T: number of available entries in the hash table
k: number of keys in the table

P(c)=λ $\lambda = \frac{k}{T}$ probability of collision by inserting the k+1 key

1-λ: probability of no collision

E[k+1]; Expected number of probes to insert the k+1 key

$$E[k+1] = 1p(1) + 2p(2) + 3p(3) + \cdots$$

$ip(i)$: product of the number of probes multiplied by the probability that key k+1 is inserted on the *i*th probe.

$$ip(i) = \lambda^{i-1}(1-\lambda)$$

$$E[k+1] = \sum_{i=1}^{\infty} i\lambda^{i-1}(1-\lambda)$$

$$= (1-\lambda)\sum_{i=1}^{\infty} i\lambda^{i-1} = (1-\lambda)\sum_{i=1}^{\infty}\frac{d}{d\lambda}(\lambda^{i}) = (1-\lambda)\frac{d}{d\lambda}\left(\sum_{i=1}^{\infty}\lambda^{i}\right)$$

$$\sum_{i=1}^{\infty}\lambda^{i} = \lim_{i\to\infty} S = \lim_{i\to\infty}\frac{1-\lambda^{i+1}}{1-\lambda} = \frac{1}{1-\lambda}$$

$$E[k+1] = (1-\lambda)\frac{d}{d\lambda}\left(\frac{1}{1-\lambda}\right) = (1-\lambda)\left(\frac{0(1-\lambda)-(-1)1}{(1-\lambda)^2}\right) = (1-\lambda)\frac{1}{(1-\lambda)^2}$$

$$E[k+1] = \frac{1}{(1-\lambda)}$$

$$E[\lambda] = \frac{1}{1-\lambda}$$

λ changes from 0 to its current value. We need to find

K : total number of keys in the table
k : number of keys in the table now.

$$I(\lambda) = \frac{1}{K}E\left[\frac{0}{T}\right] + \frac{1}{K}E\left[\frac{1}{T}\right] + \cdots + \frac{1}{K}E\left[\frac{K}{T}\right]$$

$$x = \frac{k}{T}, dx = \frac{dk}{T}, dk = Tdx$$

$$I(\lambda) \approx \frac{1}{K}\int_0^{\lambda}\frac{1}{1-x}Tdx = \frac{T}{K}\int_0^{\lambda}\frac{1}{1-x}dx = \frac{1}{\lambda}\int_0^{\lambda}\frac{1}{1-x}dx$$

$$I(\lambda) \approx -\frac{1}{\lambda}\ln(1-\lambda) + \frac{1}{\lambda}\ln(1-0) = \frac{1}{\lambda}\ln\left(\frac{1}{1-\lambda}\right)$$