AVL (Adelson, Velski, and Landis) Trees

AVL Trees are binary search trees with a balance condition.

Balance condition: For every node in the tree, the height of the left and right subtrees can differ by at most one.

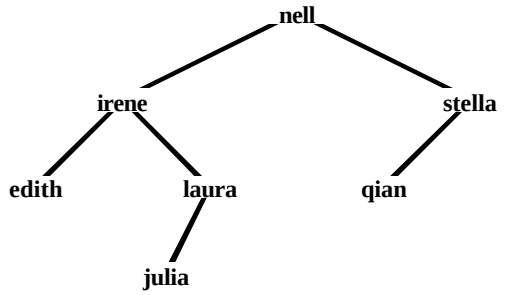

Figure 1. Binary tree satisfying the AVL balance condition

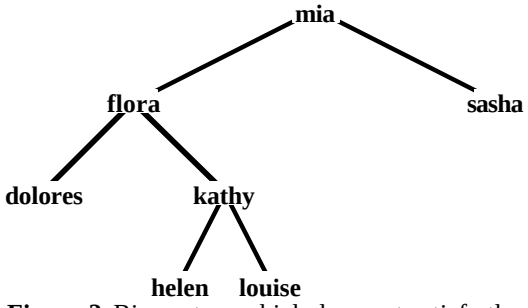

Figure 2. Binary tree which does not satisfy the AVL balance condition

Single Rotation

1. $k_1 < k_2$
2. $\{x_i \in X \mid x_i < k_1\}$
3. $\{y_i \in Y \mid k_1 < y_i < k_2$
4. $\{z_i \in Z \mid z_i > k_2\}$

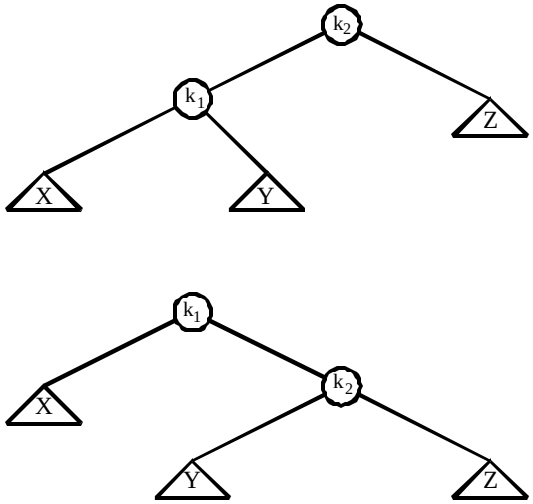

Figure 3. Clockwise rotation from top to bottom. Counter clockwise rotation from bottom to top

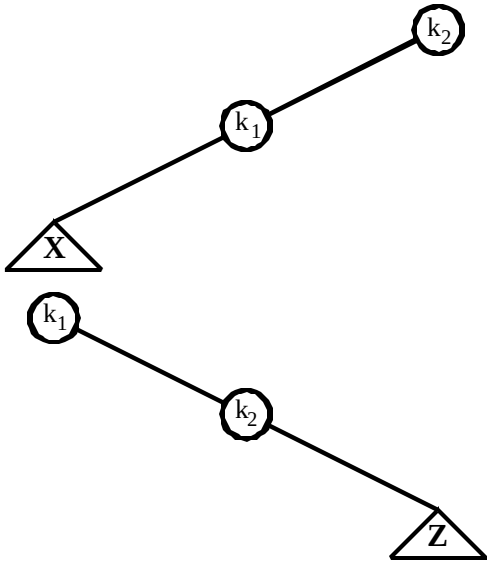

Figure 4. Recognizing a candidate for single rotation (straight legs)

Single Left Rotation example: Insert abigail, beulah, and cosette (figure 5), then balance the AVL tree (figure 6) using a single left rotation.

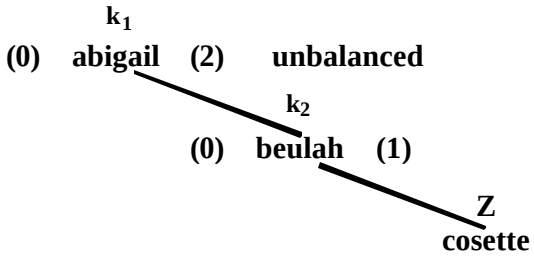

Figure 5. Unbalanced AVL tree

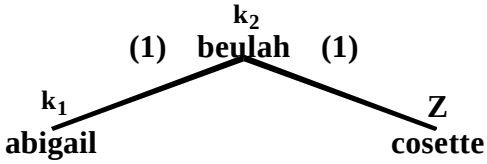

Figure 6. Balanced A VL tree (single left rotation)

Single right rotation example: Insert michelle, ilse, and deborah (figure 7), then balance the AVL tree (figure 8) using a single right rotation.

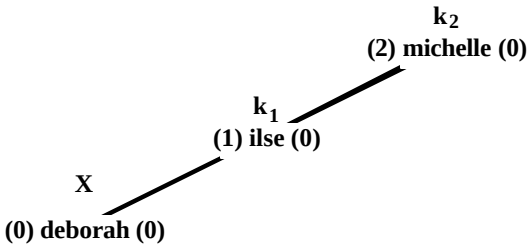

Figure 7. Unbalanced AVL tree.

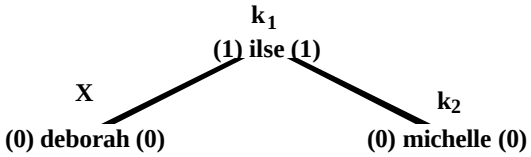

Figure 8. Balanced AVL tree (single left rotation)

Left-Right Double Rotation

1. $k_1 < k_2 < k_3$
2. $\{a_i \in A \mid a_i < k_1\}$
3. $\{b_i \in B \mid k_1 < b_i < k_2\}$
4. $\{c_i \in C \mid k_2 < c_i < k_3\}$
5. $\{d_i \in D \mid d_i > k_3\}$

The unbalanced AVL tree in figure 9 is balanced using two separate rotations. First a single left rotation is applied resulting in the tree diagramed in figure 10. Then, the tree is rotated again using a single right rotation resulting in the tree in figure 11. Together the rotations are called a left-right double rotation.

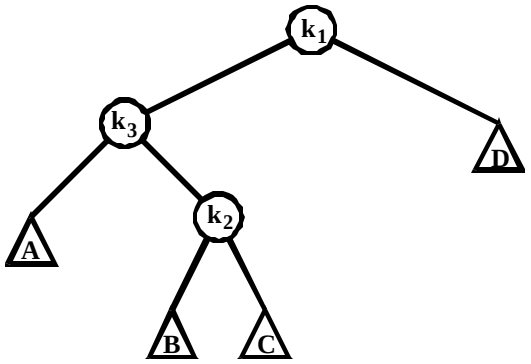

Figure 9. Unbalanced AVL Tree in need of a left-right double rotation

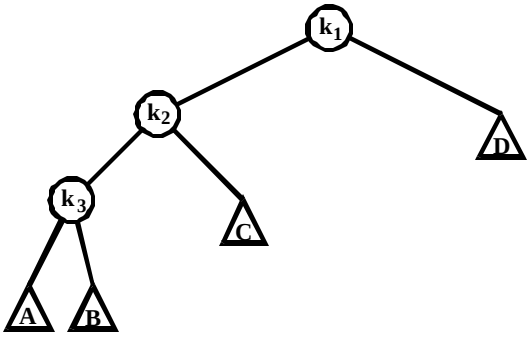

Figure 10. AVL Tree after a single left rotation

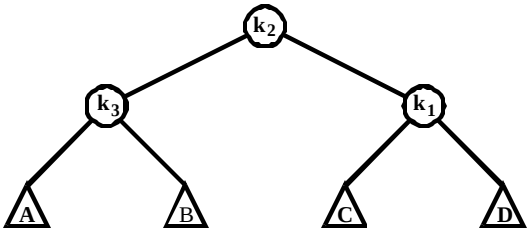

Figure 11. AVL Tree after a left-right double rotation

Right-Left Double Rotation

The unbalanced AVL tree in figure 12 is balanced using two separate rotations. First a single right rotation is applied resulting in the tree diagramed in figure 13. Then, the tree is rotated again using a single left rotation resulting in the tree in figure 14. Together the rotations are called a right-left double rotation.

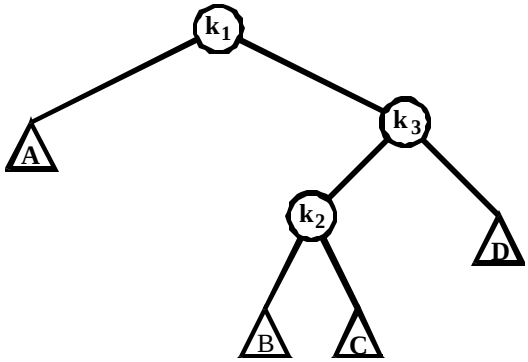

Figure 12. Unbalanced AVL tree in need of a right-left double rotation

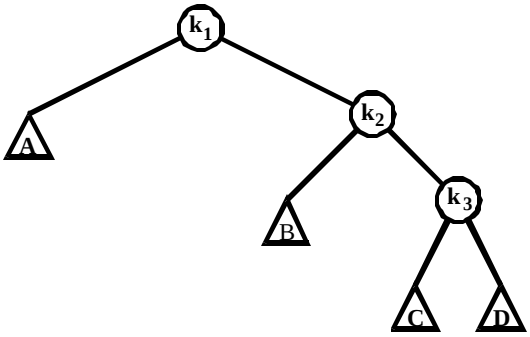

Figure 13. AVL Tree after a single right rotation

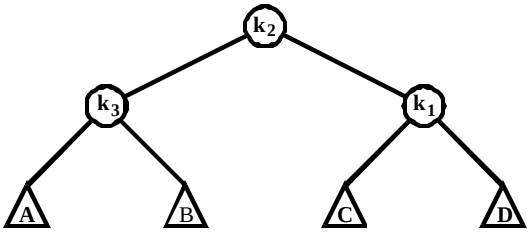

Figure 13. Balanced AVL Tree after a right-left double rotation

A candidate for a double rotation is recognized by the dogleg where the AVL tree is unbalanced. The diagram in figure 15 shows an AVL tree in need of a left-right double rotation and the diagram in figure 16 shows an AVL tree in need of a right-left double rotation.

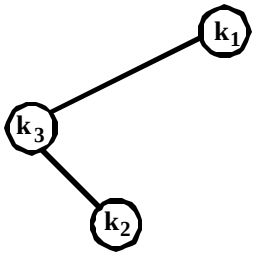

Figure 15. An AVL tree in need of a left-right double rotation

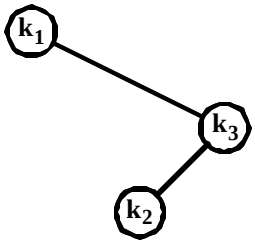

Figure 16. An AVL tree in need of a right-left double rotation

Left-right double rotation example. The diagram in figure 17 shows an AVL-tree in need of a left-right double rotation. The diagram in figure 18 shows the resulting balanced AVL tree.

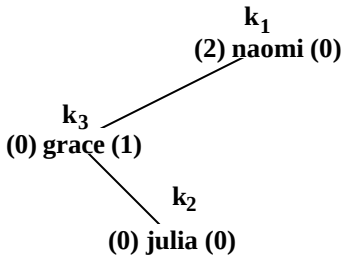

Figure 17. An AVL tree needing a left-right double rotation.

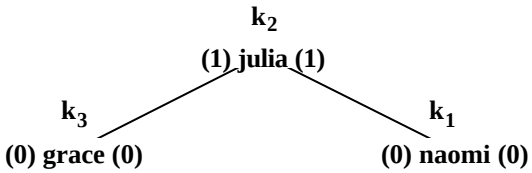

Figure 18. An AVL tree after it has been balanced by a left-right double rotation.