Kmaps – a graphical way to represent Boolean functions.

minterm

- a product term that includes all of the variables exactly once.
  Example, for a two variable Boolean function, $f(x, y)$, the minterms are $x'y'$, $x'y$, xy', and xy.
- is a row in a truth table defining a Boolean Function.
  Example: Function $F(x, y, z)$ is defined by the truth table below and the row where $x = 0$, $y = 0$, and $z = 0$ is one of eight minterms in this table represented as $x'y'z'$.

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- Each minterm is associated with a binary value and its corresponding decimal equivalent. The Boolean variables are ordered, first x, then y. Find binary values for each term such that the term, evaluated as a Boolean expression, produces a one (1). Select $x = 0$ and $y = 0$ in order to make the term $x'y' = 1$. The binary value of $x'y'$ is 00 or 0 decimal. In a similar way, one can find values for the terms $x'y = 01$ or 1 decimal, $xy' = 10$ or 2 decimal, and $xy = 11$ or 3 decimal.

m_i	x	y	Minterm	x	y
m_0	x' 0	y' 0	$x'y'$	0	0
m_1	x' 0	y 1	$x'y$	0	1
m_2	x 1	y' 0	xy'	1	0
m_3	x 1	y 1	xy	1	1

FIGURE 3.9
Minterms for Two Variables

Given three input Boolean variables, say x, y, and z, there are eight minterms. $x'y'z'$, $x'y'z$, $x'yz'$, $x'yz$, $xy'z'$, $xy'z$, $x'y'z$, and xyz. The decimal values associated with the foregoing minterms are 0, 1, 2, 3, 4, 5, 6, and 7 respectively.

m_i	x	y	z	Minterm	x	y	z
m_0	x' 0	y' 0	z' 0	$x'y'z'$	0	0	0
m_1	x' 0	y' 0	z 1	$x'y'z$	0	0	1
m_2	x' 0	y 1	z' 0	$x'yz'$	0	1	0
m_3	x' 0	y 1	z 1	$x'yz$	0	1	1
m_4	x 1	y' 0	z' 0	$xy'z'$	1	0	0
m_5	x 1	y' 0	z 1	$xy'z$	1	0	1
m_6	x 1	y 1	z' 0	xyz'	1	1	0
m_7	x 1	y 1	z 1	xyz	1	1	1

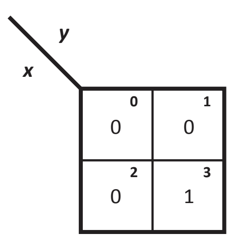
FIGURE 3.10
Minterms for Three Variables

A Kmap

- is a truth table
- is a table with a cell for each minterm.

EXAMPLE 3.10 $F(x, y) = xy$

m_i	x	y	$F(x, y) = xy$
m_0	0	0	0
m_1	0	1	0
m_2	1	0	0
m_3	1	1	1

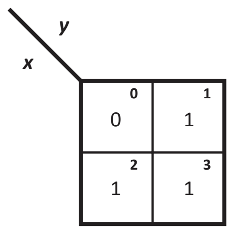


Kmap for $F(x, y) = xy = \sum(3)$

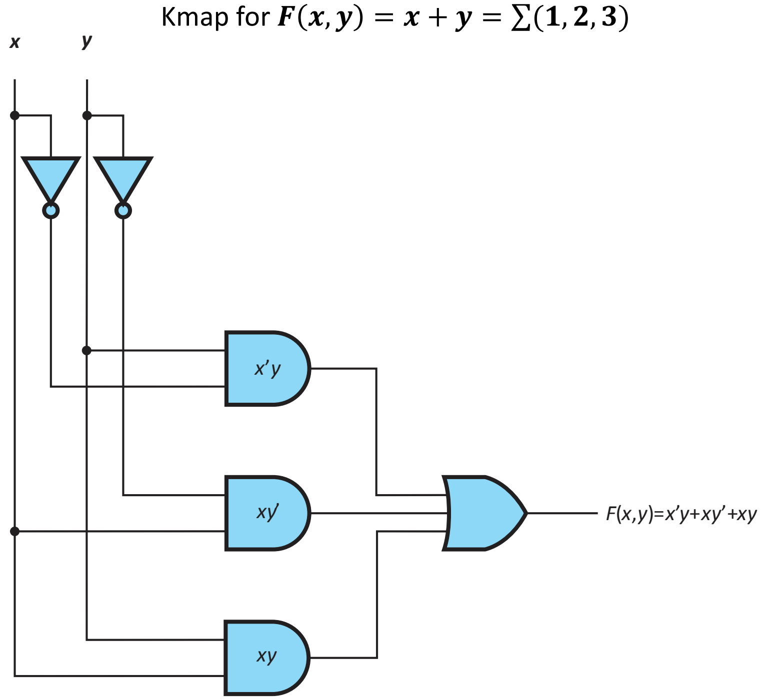
Please note that the minterm numbers – the cell numbers – are placed in the upper right of each cell. The values printed in each cell are the values of the Boolean function $F(x, y)$.

EXAMPLE 3.11 $F(x, y) = x + y$

m_i	x	y	$F(x, y) = xy$
m_0	0	0	0
m_1	0	1	1
m_2	1	0	1
m_3	1	1	1



Kmap for $\boldsymbol{F(x, y) = x + y = \sum(1, 2, 3)}$

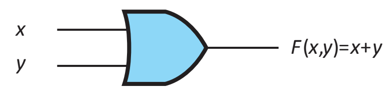


$F(x, y) = x'y + xy' + xy$
Not minimized

Please note that the minterm numbers – the cell numbers – are placed in the upper right of each cell. The values printed in each cell are the values of the Boolean function $F(x, y)$.

Three of the minterms in Example 3.11 have a value of 1, exactly the minterms for which the input to the function gives us a 1 for the output. To assign 1s in the Kmap, we simply place 1s where we find corresponding 1s in the truth table.

$F(x, y)$	$=$	$x'y + xy' + xy$	Original expression
	$=$	$x'y + xy' + xy + xy$	Idempotent Law (OR Form)
	$=$	$x'y + xy + xy' + xy$	Commutative Law (OR Form)
	$=$	$(x' + x)y + x(y' + y)$	Distributive Law (OR Form) applied twice
	$=$	$(1)y + x(1)$	Inverse Law (OR Form) applied twice
	$=$	$y + x$	Identity Law (AND Form) applied twice
	$=$	$x + y$	Commutative Law (OR Form)



$F(x, y) = x + y$
Minimized

MAXTERM

Maxterms are an alternative way of identifying the rows in a truth table. Maxterms are the complements of minterms.

$M_0 = \overline{m_0}$	$M_0 = \overline{x'y'z'}$	$M_0 = x + y + z$
$M_1 = \overline{m_1}$	$M_1 = \overline{x'y'z}$	$M_1 = x + y + z'$
$M_2 = \overline{m_2}$	$M_2 = \overline{x'yz'}$	$M_2 = x + y' + z$
$M_3 = \overline{m_3}$	$M_3 = \overline{x'yz}$	$M_3 = x + y' + z'$
$M_4 = \overline{m_4}$	$M_4 = \overline{xy'z'}$	$M_4 = x' + y + z$
$M_5 = \overline{m_5}$	$M_5 = \overline{xy'z}$	$M_5 = x' + y + z'$
$M_6 = \overline{m_6}$	$M_6 = \overline{xyz'}$	$M_6 = x' + y' + z$
$M_7 = \overline{m_7}$	$M_7 = \overline{xyz}$	$M_7 = x' + y' + z'$

Continuing our example and recalling that $F(x, y) = \sum(1,2,3)$, we observe that $\overline{F} = \sum(0) = \overline{x'y'}$. We find $\overline{F}$ by finding all the rows where a zero (0) was entered in the output column.
To find F, using our complement, we complement $\overline{F}$.

$$F = \overline{\overline{F}} = F = M_0 = \prod(0) = \overline{x'y'} = x + y$$

Once the minterms for a function are known, we can immediately express the same function as a product of maxterms.
Example:
Given

$$F(x, y) = \sum(1,2,3)$$

we know immediately that

$$F(x, y) = \prod(0)$$

In summary, we can find a function F as the sum of products – the sum of those minterms where a one (1) has been entered in the output column or we can find function F as the product of sums – the product of those maxterms where a zero has been entered in the output column.

The Kmap will assist us to minimize the expressions for Boolean functions and, thereby, minimize the number of gates needed to implement the function.

In class exercise: Express $F(x, y, z)$ in canonical SOP form given its truth table below.

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Solution: $F(x, y, z) = \sum(1,2,4,7) = x'y'z + x'yz' + xy'z' + xyz$

In class exercise: Express $F(x, y, z)$ in canonical POS form given its truth table below.

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Solution: $F(x,y,z) = F''(x,y,z) = (\sum(0,3,5,6))' = (x'y'z' + x'yz + xy'z + xyz')$

$$F(x,y,z) = \prod(0,3,5,6) = (x + y + z)(x + y' + z')(x' + y + z')(x' + y' + z)$$

$$(0 + 0 + 1)(0 + 1 + 0)(1 + 0 + 0)(1 + 1 + 1) = 1$$

In class exercise: Draw the truth table for the function $F(x,y,z) = x'y'z + x'yz' + xyz$.

Solution: $F(x, y, z) = x'y'z + x'yz' + xyz = \sum(1,2,7)$

1. Draw a 3-variable truth table including designations for the minterms.

m_i	x	y	z	F
m_0	0	0	0	
m_1	0	0	1	
m_2	0	1	0	
m_3	0	1	1	
m_4	1	0	0	
m_5	1	0	1	
m_6	1	1	0	
m_7	1	1	1	

2. Mark the minterms in the output column F where a one (1) is placed – in rows 1, 2, and 7.

m_i	x	y	z	F
m_0	0	0	0	
m_1	0	0	1	1
m_2	0	1	0	1
m_3	0	1	1	
m_4	1	0	0	
m_5	1	0	1	
m_6	1	1	0	
m_7	1	1	1	1

3. Place zeros in the remaining rows..

m_i	x	y	z	F
m_0	0	0	0	0
m_1	0	0	1	1
m_2	0	1	0	1
m_3	0	1	1	0
m_4	1	0	0	0
m_5	1	0	1	0
m_6	1	1	0	0
m_7	1	1	1	1