Two common representations for Boolean functions

- sum-of-products (SOP) – requires that the expression be a collection of ANDed variables (or product terms) that are ORed together.
  Examples:

$F_1(x,y,z) = xy + yz' + xyz$	Yes – in SOP form
$F_2(x,y,z) = xy' + x(y + z')$	No – not in SOP form

- product-of-sums (POS) – requires that the expression be a collection of terms that are ORed together (sum terms) and that the Boolean function is a product of all the sum terms.
  Examples:

$F_1(x,y,z) = (x + y)(x + z')(y + z')(y + z)$	Yes – in POS form
$F_2(x,y,z) = y(x'z + xz') + x(yz + yz')$	No – not in SOP form

Example 3.9 Consider a simple majority function. This is a function that, when given three inputs, outputs a 0 if less than half of its inputs are 1, and a 1 if at least half of its input are 1. Table 3.8 depicts the truth table for this majority function.

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

MINTERM

A *minterm*, m_i, is row in the truth table. The rows having a one (1) in the output column, summed together completely specify the output.

Example: Consider the truth-table below.

	m_i	x	y	z		F
x'y'z'	0	0	0	0		0
x'y'z	1	0	0	1		0
x'yz'	2	0	1	0		0
x'yz	3	0	1	1		1
x'yz	4	1	0	0		0
xy'z	5	1	0	1		1
xyz'	6	1	1	0		1
xyz	7	1	1	1		1

Row 0, for example, where $x = 0, y = 0$, and $z = 0$, is called *minterm* zero or m_0.
We use summation notation to represent function F as the sum of minterms.

$$F = \sum(3,5,6,7) = m_3 + m_5 + m_6 + m_7$$

We can express function F in several ways.

$F(x,y,z) = x'yz + xy'z + xyz' + xyz$
$F(x,y,z) = 011 + 101 + 110 + 111$
$F(x,y,z) = m_3 + m_5 + m_6 + m_7$
$F = \sum\big((3,5,6,7)\big)$

Minterms are expressed as the sum of products (SOP).

MAXTERM

Maxterms are an alternative way of identifying the rows in a truth table. Maxterms are the complements of minterms.

$M_0 = \overline{m_0}$	$M_0 = \overline{x'y'z'}$	$M_0 = x + y + z$
$M_1 = \overline{m_1}$	$M_1 = \overline{x'y'z}$	$M_1 = x + y + z'$
$M_2 = \overline{m_2}$	$M_2 = \overline{x'yz'}$	$M_2 = x + y' + z$
$M_3 = \overline{m_3}$	$M_3 = \overline{x'yz}$	$M_3 = x + y' + z'$
$M_4 = \overline{m_4}$	$M_4 = \overline{xy'z'}$	$M_4 = x' + y + z$
$M_5 = \overline{m_5}$	$M_5 = \overline{xy'z}$	$M_5 = x' + y + z'$
$M_6 = \overline{m_6}$	$M_6 = \overline{xyz'}$	$M_6 = x' + y' + z$
$M_7 = \overline{m_7}$	$M_7 = \overline{xyz}$	$M_7 = x' + y' + z'$

Continuing our example and recalling that $F(x,y,z) = \sum(3,5,6,7)$, we observe that $\overline{F} = \sum(0,1,2,4) = x'y'z' + x'y'z + x'yz' + xy'z'$. We find $\overline{F}$ by finding all the rows where a zero (0) was entered in the output column.

To find F, using our complement, we complement $\overline{F}$.

$$F = \overline{\overline{F}} = \overline{x'y'z' + x'y'z + x'yz' + xy'z'} = (x + y + z)(x + y + z')(x + y' + z)(x' + y + z)$$

$$F = M_0 M_1 M_2 M_4 = \prod(0,1,2,4)$$

Once the minterms for a function are known, we can immediately express the same function as a product of maxterms.

Example:

Given

$$F(x,y,z) = \sum(3,5,6,7)$$

we know immediately that

$$F(x,y,z) = \prod(0,1,2,4)$$

In summary, we can find a function F as the sum of products – the sum of those minterms where a one (1) has been entered in the output column or we can find function F as the product of sums – the product of those maxterms where a zero has been entered in the output column.