

Applying the NOT operator to a Boolean function produces the function's complement.

Example:

$$F(x, y, z) = (x + y + z)$$

The complement is expressed as $F'(x, y, z)$

$$F'(x, y, z) = (x + y + z)'$$

$$\text{Let } w = (x + y)$$

$$\text{Then } F'(x, y, z) = (w + z)'$$

and $(w + z)' = w'z'$ by DeMorgan's Law

Now $w' = (x + y)' = x'y'$ again by DeMorgan's Law

We can now say that $F'(x, y, z) = (x + y + z)' = x'y'z'$

In general, when we complement a function, each Boolean variable is complemented and the operators are also complemented – that is – every $+$ is replaced by a \cdot and every \cdot is replaced by a $+$.

3. Using DeMorgan's Law, write an expression for the complement of F if $F(x, y, z) = xy'(x + z)$.

$$\begin{aligned} F(x, y, z) &= xy'(x + z) \\ F'(x, y, z) &= (xy'(x + z))' \\ &= (xy')' + (x + z)' && \text{DeMorgan's Law - AND Form} \\ &= x' + y'' + (x + z)' && \text{DeMorgan's Law - AND Form} \\ &= x' + y + (x + z)' && \text{Double Complement Law} \\ &= x' + y + x'z' && \text{DeMorgan's Law - OR Form} \\ &= x' + x'z' + y && \text{Commutative Law - OR Form} \\ &= x'(1 + z') + y && \text{Distributive Law - OR Form} \\ &= x'(1) + y && \text{Null (or Dominance) Law - OR Form} \\ &= x' + y && \text{Identity Law - AND Form} \end{aligned}$$