

Applying the NOT operator to a Boolean function produces the function's complement.

Example:

$$F(x, y, z) = (x + y + z)$$

The complement is expressed as $F'(x, y, z)$

$$F'(x, y, z) = (x + y + z)'$$

$$\text{Let } w = (x + y)$$

$$\text{Then } F'(x, y, z) = (w + z)'$$

$$\text{and } (w + z)' = w'z' \text{ by DeMorgan's Law}$$

$$\text{Now } w' = (x + y)' = x'y' \text{ again by DeMorgan's Law}$$

$$\text{We can now say that } F'(x, y, z) = (x + y + z)' = x'y'z'$$

In general, when we complement a function, each Boolean variable is complemented and the operators are also complemented – that is – every + is replaced by a · and every · is replaced by a +.

3. Using DeMorgan's Law, write an expression for the complement of F if $F(x, y, z) = xy'(x + z)$.

$F(x, y, z)$	$=$	$xy'(x + z)$	
$F'(x, y, z)$	$=$	$(xy'(x + z))'$	
	$=$	$(xy')' + (x + z)'$	DeMorgan's Law – AND Form
	$=$	$x' + y'' + (x + z)'$	DeMorgan's Law – AND Form
	$=$	$x' + y + (x + z)'$	Double Complement Law
	$=$	$x' + y + x'z'$	DeMorgan's Law – OR Form
	$=$	$x' + x'z' + y$	Commutative Law – OR Form
	$=$	$x'(1 + z') + y$	Distributive Law – OR Form
	$=$	$x'(1) + y$	Null (or Dominance) Law – OR Form
	$=$	$x' + y$	Identity Law – AND Form