Example 3.1

$F(x,y)$	$=$	$xy + xy$	Original Statement
	$=$	xy	Idempotent Law – OR Form $x + x = x$

Example 3.2 Given the function $F(x,y,z) = x'yz + x'yz' + xz$, we simplify as follows

$F(x,y,z)$	$=$	$x'yz + x'yz' + xz$	Original Statement
	$=$	$x'y(z + z') + xz$	Distributive Law – OR Form
	$=$	$x'y(1) + xz$	Inverse Law – OR Form
	$=$	$x'y + xz$	Identity Law – AND Form

Example 3.3 Given the function $F(x,y,z) = y + (xy)'$, we simplify as follows

$F(x,y,z)$	$=$	$y + (xy)'$	Original Statement
	$=$	$y + x' + y'$	DeMorgan's Law – AND Form
	$=$	$y + y' + x'$	Commutative Law – OR Form
	$=$	$1 + x'$	Inverse Law – OR Form
	$=$	1	Null (or Dominance) Law – OR Form

Example 3.4 Given the function $F(x,y,z) = (xy)'(x' + y)(y' + y)$, we simplify as follows

$F(x,y,z)$	$=$	$(xy)'(x' + y)(y' + y)$	Original Statement
	$=$	$(xy)'(x' + y)(1)$	Inverse Law – OR Form
	$=$	$(xy)'(x' + y)$	Identity Law – AND Form
	$=$	$(x' + y')(x' + y)$	DeMorgan's Law – AND Form
	$=$	$x' + y'y$	Distributive Law – AND Form
	$=$	$x' + 0$	Inverse Law – AND Form
	$=$	x'	Identity Law – OR Form

Practice Given the function $F(x,y,z) = y(x' + (x + y)')$,we simplify as follows

$F(x,y,z)$	$=$	$y(x' + (x + y)')$	Original Statement
	$=$		
	$=$		
	$=$		
	$=$		
	$=$		
	$=$		

Practice Given the function $F(x, y, z) = y(x' + (x + y)')$,we simplify as follows

$F(x, y, z)$	$=$	$y(x' + (x + y)')$	Original Statement
	$=$	$y(x' + x'y')$	DeMorgan's Law – OR Form
	$=$	$y(x'(1 + y'))$	Distributive Law – OR Form
	$=$	$y(x'(1))$	Null (or Dominance) Law – OR Form
	$=$	$y(x')$	Identity Law – AND Form
	$=$	$x'y$	Commutative Law – AND Form

Trickier Example 3.5 Given the function $F(x, y, z) = x'(x + y) + (y + x)(x + y')$, we simplify as follows

$F(x, y, z)$	$=$	$x'(x + y) + (y + x)(x + y')$	Original Statement
	$=$	$x'(x + y) + (x + y)(x + y')$	Commutative Law – OR Form
	$=$	$x'(x + y) + (x + yy')$	Distributive Law – AND Form
	$=$	$x'(x + y) + (x + 0)$	Inverse Law – AND Form
	$=$	$x'(x + y) + x$	Identity Law – OR Form
	$=$	$x'x + x'y + x$	Distributive Law – OR Form
	$=$	$0 + x'y + x$	Inverse Law – AND Form
	$=$	$x'y + x$	Identity Law – OR Form
	$=$	$x + x'y$	Commutative Law – OR Form
	$=$	$(x + x')(x + y)$	Distributive Law – AND Form
	=	$(1)(x + y)$	Inverse Law – OR Form
	=	$x + y$	Identity Law – AND Form

Practice Given the function $F(x, y, z) = xy'z + x(y + z')' + xy'z'$, we simplify as follows

$F(x, y, z)$	$=$	$xy'z + x(y + z')' + xy'z'$	Original Statement
	$=$		
	$=$		
	$=$		
	$=$		
	$=$		
	$=$		
	$=$		
	$=$		
	$=$		
	$=$		

Practice Given the function $F(x,y,z) = xy'z + x(y+z')' + xy'z'$, we simplify as follows

$F(x,y,z)$	$=$	$xy'z + x(y+z')' + xy'z'$	Original Statement
	$=$	$xy'z + xy'z' + x(y+z')'$	Commutative Law – OR Form
	$=$	$xy'(z+z') + x(y+z')'$	Distributive Law – OR Form
	$=$	$xy'(1) + x(y+z')'$	Inverse Law – OR Form
	$=$	$xy' + x(y+z')'$	Identity Law – AND Form
	$=$	$xy' + x(y'z)$	DeMorgan's Law – OR Form
	$=$	$xy'(1+z)$	Distributive Law – OR Form
	$=$	$xy'(1)$	Null (or Dominance) Law – OR Form
	$=$	xy'	Identity Law – AND Form

Trickier Example 3.6 Given the function $F(x, y, z) = xy + x'z + yz$, we simplify as follows

$F(x, y, z)$	$=$	$xy + x'z + yz$	Original Statement
	$=$	$xy + x'z + yz(1)$	Identity Law – AND Form
	$=$	$xy + x'z + yz(x + x')$	Inverse Law – OR Form
	$=$	$xy + x'z + (yz)x + (yz)x'$	Distributive Law – OR Form
	$=$	$xy + x'z + x(yz) + x'(yz)$	Commutative Law – AND Form (twice)
	$=$	$xy + x'z + x(yz) + x'(zy)$	Commutative Law – AND Form
	$=$	$xy + x'z + (xy)z + (x'z)y$	Associative Law – AND Form (twice)
	$=$	$xy + (xy)z + x'z + (x'z)y$	Commutative Law – OR Form
	$=$	$xy(1 + z) + x'z(1 + y)$	Distributive Law – OR Form (twice)
	$=$	$xy(1) + x'z(1)$	Null (or Dominance) Law – OR Form (twice)
	$=$	$xy + x'z$	Identity Law – AND Form (twice)

Example 3.7 Prove that $(x + y)(x' + y) = y$.

$(x + y)(x' + y)$	$=$	$xx' + xy + yx' + yy$	Distributive Law – OR Form (twice)
	$=$	$0 + xy + yx' + yy$	Inverse Law – AND Form
	$=$	$0 + xy + yx' + y$	Idempotent Law – AND Form
	$=$	$xy + yx' + y$	Identity Law – OR Form
	$=$	$yx + yx' + y$	Commutative Law – AND Form
	$=$	$y(x + x') + y$	Distributive Law – OR Form
	$=$	$y(1) + y$	Inverse Law – OR Form
	$=$	$y + y$	Identity Law – AND Form
	$=$	y	Idempotent Law – OR Form

Prove DeMorgan's Law – OR Form $(x + y)' = x'y'$ by means of a truth table.

1. Create column titles – setting up the proof. There are four (4) rows, one for each of the possible combinations for the values of x and y.

x	y	$x + y$	$(x + y)'$	$=$	$x'y'$	x'	y'

2. Fill in values for y.

x	y	$x + y$	$(x + y)'$	$=$	$x'y'$	x'	y'
	0						
	1						
	0						
	1						

3. Fill in complementary values for y'.

x	y	$x + y$	$(x + y)'$	$=$	$x'y'$	x'	y'
	0						1
	1						0
	0						1
	1						0

4. Fill in values for x so that each row has a unique combination of x and y and so that all combinations are listed.

x	y	$x+y$	$(x+y)'$	$=$	$x'y'$	x'	y'
0	0						1
0	1						0
1	0						1
1	1						0

5. Fill in complementary values for x'.

x	y	$x+y$	$(x+y)'$	$=$	$x'y'$	x'	y'
0	0					1	1
0	1					1	0
1	0					0	1
1	1					0	0

6. Compute values for $x + y$.

x	y	$x+y$	$(x+y)'$	$=$	$x'y'$	x'	y'
0	0	0				1	1
0	1	1				1	0
1	0	1				0	1
1	1	1				0	0

7. Compute values for $(x + y)'$.

x	y	$x+y$	$(x+y)'$	$=$	$x'y'$	x'	y'
0	0	0	1			1	1
0	1	1	0			1	0
1	0	1	0			0	1
1	1	1	0			0	0

8. Compute values for $x'y'$.

x	y	$x+y$	$(x+y)'$	$=$	$x'y'$	x'	y'
0	0	0	1		1	1	1
0	1	1	0		0	1	0
1	0	1	0		0	0	1
1	1	1	0		0	0	0

9. Note that for every combination of x and y – that is all possible values of x and y that $(x + y)' = x'y'$.

x	y	$x+y$	$(x+y)'$	$=$	$x'y'$	x'	y'
0	0	0	1	1	1	1	1
0	1	1	0	1	0	1	0
1	0	1	0	1	0	0	1
1	1	1	0	1	0	0	0