2.3.1 Converting from a foreign base to decimal

$$(N)_r = \sum_{j=-m}^{n-1} a_j r^j = a_{n-1}r^{n-1} + a_{n-2}r^{n-2} + \cdots + a_0 r^0 + a_{-1}r^{-1} + a_{-2}r^{-2} + \cdots a_{-m}r^{-m}$$

Example:

$(1976.4)_{10}$	=	**1**	$\times$	**10^3**	=	**1000**
	+	**9**	$\times$	**10^2**	=	**900**
	+	**7**	$\times$	**10^1**	=	**70**
	+	**6**	$\times$	**10^0**	=	**6**
	+	**4**	$\times$	**10^{-1}**	=	**4/10**
						1976 4/10

$(2bad.a)_{16}$	=	**2**	$\times$	**16^3**	=	**8192**
	+	**11 (b)**	$\times$	**16^2**	=	**2816**
	+	**10 (a)**	$\times$	**16^1**	=	**160**
	+	**13 (d)**	$\times$	**16^0**	=	**13**
	+	**10 (a)**	$\times$	**16^{-1}**	=	**4/10**
						11181.625

$$(((2\times16)+11\times16)+10\times16)+13=11{,}181$$

Example:

$(1101.101)_2$	=	**1**	$\times$	**2^3**	=	**8**
	+	**1**	$\times$	**2^2**	=	**4**
	+	**0**	$\times$	**2^1**	=	**0**
	+	**1**	$\times$	**2^0**	=	**1**
	+	**1**	$\times$	**2^{-1}**	=	**½**
	+	**0**	$\times$	**2^{-2}**	=	**0**
	+	**1**	$\times$	**2^{-3}**	=	**1/8**
						13 5/8

Example:

$(test.i)_{36}$	=	**29 (t)**	$\times$	**36^3**	=	**1,353,024**
	+	**14 (e)**	$\times$	**36^2**	=	**18,144**
	+	**28 (s)**	$\times$	**36^1**	=	**1,008**
	+	**29 (t)**	$\times$	**36^0**	=	**29**
	+	**18 (i)**	$\times$	**36^{-1}**	=	**18/36**
						1,372,205.5

2.3.2 Converting from decimal to a foreign base

The Radix Divide/Multiply Method

Objective:	Covert a decimal number to an equivalent number in another radix (base).	
Solution:	**Step**	**Discussion**
	1	Separate the integer and fractional portions of the decimal number
	Integer portion conversion algorithm:	
	Step	**Discussion**
	0	Divide the decimal number by the radix. The remainder is a_i, i=0,1,2,3 … *n*-1. Quotient q_i becomes the dividend in the iteration. r : radix, divisor d_i : dividend in iteration *i*. d_0 is the initial dividend, the decimal number to be converted to the foreign base. q_i : quotient in iteration *i* a_i : remainder produced in iteration *i*, *i*th digit of the number in radix *r*. $a_i = d_i \bmod r, d_{i+i} = \lfloor d_i \div r \rfloor$
	1 … *n*-1	Perform step 0 until the dividend equal zero.

Example: Convert 29_{10} to binary.

Step	Radix-Divisor	Dividend	Quotient	Remainder	a_i
0	2	29	14	1	a_0
1	2	14	7	0	a_1
2	2	7	3	1	a_2
3	2	3	1	1	a_3
4	2	1	0	1	a_4
5	2	0 **stop!**			

$29_{10} = 11101_2$

11101_2	**=**		**1**	$\times$	**2^4**	**=**	**16**
		+	**1**	$\times$	**2^3**	**=**	**8**
		+	**1**	$\times$	**2^2**	**=**	**4**
		+	**0**	$\times$	**2^1**	**=**	**0**
		+	**1**	$\times$	**2^0**	**=**	**1**
							29

	Fractional portion conversion algorithm:	
	Step	**Discussion**
	-1	Multiply the decimal fraction by the radix. The integer portion of the product is a_{-i}, i=-1,-2, … -m
	-2 … *-m*	Remove the integer portion of the product obtained in the previous step. The remaining decimal fraction is the multiplicand for this step. $\text{fraction}_{i+1}=\text{frac}(\text{product}_i)$ Repeat step –1 and the foregoing until either $\text{fraction}_{i+1}=0$ or the desired precision is obtained.

Example: Convert 20.1_{10} to binary.

Step	**Radix-Multiplier**	**Fraction-Multiplicand**	**Product**	**Integer Portion**	a_i
-1	2	0.1	0.2	0	a_{-1}
-2	2	0.2	0.4	0	a_{-2}
-3	2	0.4	0.8	0	a_{-3}
-4	2	0.8	1.6	1	a_{-4}
-5	2	0.6	1.2	1	a_{-5}
-6	2	0.2	0.4	0	a_{-6}
-7	2	0.4	0.8	0	a_{-7}
-8	2	0.8	1.6	1	a_{-8}

$$0.1_{10} \approx 0.00011001_2$$

0.00011001_2	**=**		**0**	$\times$	**2^{-1}**	**=**	
		+	**0**	$\times$	**2^{-2}**	**=**	
		+	**0**	$\times$	**2^{-3}**	**=**	
		+	**1**	$\times$	**2^{-4}**	**=**	**1/16**
		+	**1**	$\times$	**2^{-5}**	**=**	**1/32**
		+	**0**	$\times$	**2^{-6}**	**=**	
		+	**0**	$\times$	**2^{-7}**	**=**	
		+	**1**	$\times$	**2^{-8}**	**=**	**1/256**
							0.09765625

Practice Problems.

1. Convert 38.65_{10} to binary.
2. Convert 42_5 to $\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\__7$.