1. Print your name on your scantron in the space labeled **NAME**.
2. Print **CMSC 2123** in the space labeled **SUBJECT**.
3. Print the date **5-8-2019**, in the space labeled **DATE**.
4. Print your **CRN** in the space labeled **PERIOD**.
   4.1. Print **20936** in the space labeled **PERIOD** if you are enrolled in the **on-campus** section.
   4.2. Print **22209** in the space labeled **PERIOD** if you are enrolled in the **IVE** section.
5. Print the test number and version, **T3/V1**, in the space labeled **TEST NO.**
6. This is a closed-book examination. No reference materials are permitted. No notes are permitted.
7. You may not consult your neighbors, colleagues, or fellow students to answer the questions on this test.
8. Cellular phones are prohibited. The possessor of a cellular phone will receive a ***zero*** (***0***) if the phone rings or is visible during the test.
9. You may use your personal calculator on this test. You are prohibited from loaning your calculator or borrowing a calculator from another person enrolled in this course.
10. Mark the best selection that satisfies the question. If selection **b** is better that selections **a** and **d**, then mark selection **b**. Mark only ***one*** selection.
11. Darken your selections completely. Make a heavy black mark that completely fills your selection.
12. Answer all **50** questions.
13. Record your answers on SCANTRON form **882-E (It is green!)**
14. When you have completed the test, place your scantron, face up, between pages 2 and 3 of your questionnaire and submit both the questionnaire and your scantron to your instructor.

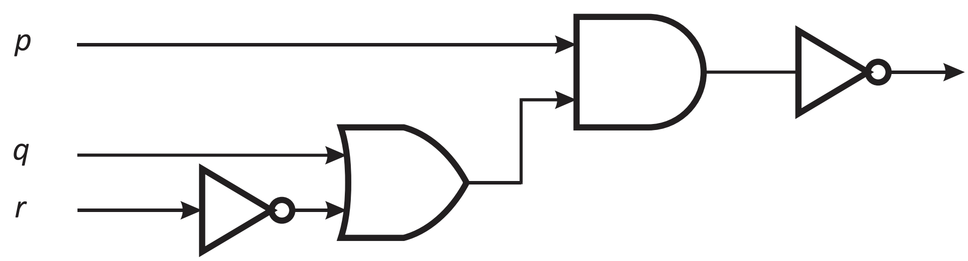
1. (1.1 Propositional Logic) Let p and q be the propositions "The election is decided" and "The votes have been counted," respectively. Which compound proposition is **NOT** expressed properly as an English sentence?

	Proposition	**Sentence**
a.	$\neg p \vee q$	The election is not decided, or the votes have been counted.
b.	$\neg p \rightarrow \neg q$	If the votes have not been counted, then the election is decided.
c.	$p \leftrightarrow q$	The election is decided if and only if the votes have been counted.
d.	$\neg q \vee (\neg p \wedge q)$	Either the votes have not been counted, or else the election is not decided and the votes have been counted.

2. (1.1 Propositional Logic) Determine which of these compound propositions are true.

   a. $5 + 2 = 8$ if and only if $7 - 3 = 5$.
   b. if $1 + 2 = 3$, then $2 + 3 = 4$.
   c. $2 + 2 = 5$ or $1 - 3 = 2$.
   d. $2 + 1 = 4$ and $3 - 1 = 2$.

3. (1.2 Applications of Propositional Logic) Find the output of the combinatorial circuit shown.



   a. $\neg q \vee r \;\; \wedge \neg p$
   b. $q \wedge \neg r \;\; \vee \neg p$
   c. none
   d. $\neg q \wedge r \;\; \vee \neg p$

4. (1.3 Propositional Equivalences) Which of the following statements is true.

   a. $[(p \rightarrow q) \wedge p] \rightarrow (p \wedge q)$ is a tautology.
   b. $[(p \rightarrow q) \wedge p] \rightarrow \neg p$ is a contradiction.
   c. $[(p \rightarrow q) \wedge p] \rightarrow q$ is a contingency.
   d. $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are logically equivalent.

5. (1.4 Predicates and Quantifiers) Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Let the domain consist of a students in your class. Which statement below accurately represents the statement "For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet."

   a. $\forall x(C(x) \lor D(x) \lor D(x))$
   b. $(\exists x(C(x)) \land (\exists x(D(x)) \land (\exists x(F(x))$
   c. $(\exists x(C(x)) \lor (\exists x(D(x)) \lor (\exists x(F(x))$
   d. $\forall x(x \rightarrow C(x) \land D(x) \land F(x))$

6. (1.4 Predicates and Quantifiers) Let $C(x)$ be the statement "x is a comedian" and let $F(x)$ be the statement "x is funny." Select the correct translation of the expression $\exists x(C(x) \land F(x))$.
   a. Every comedian is funny.
   b. Every person is a funny comedian.
   c. There exists a person such that if she or he is a comedian, then she or he is funny.
   d. Some comedians are funny.

7. (1.5 Nested Quantifiers) Let $I(x)$ be the statement "x has an Internet connection" and $C(x, y)$ be the statement "x and y have chatted over the Internet," where the domain for the variables x and y consists of all students in your class. Select the expression that is the translation of the statement "Everyone in your class with an Internet connection has chatted over the Internet with at least one other student in your class."

   a. $\exists x(I(x) \land \forall y(x \neq y \rightarrow C(x, y)))$
   b. $\forall x(I(x) \leftrightarrow \exists y(x \neq y \land C(x, y)))$
   c. $\forall x(I(x) \rightarrow \exists y(x \neq y \land C(x, y)))$
   d. $\exists x(I(x) \lor \forall y(x \neq y \rightarrow C(x, y)))$

8. (1.5 Nested Quantifiers) Select the alternative where the statement shown was rewritten so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

$$\neg(\exists x \exists y \neg P(x, y) \land \forall x \forall y Q(x, y))$$

   a. $(\forall x \forall y P(x, y)) \land (\exists x \exists y \neg Q(x, y)))$
   b. $(\exists x \exists y P(x, y)) \lor (\forall x \forall y \neg Q(x, y)))$
   c. $(\forall x \forall y P(x, y)) \lor (\exists x \exists y \neg Q(x, y)))$
   d. $(\exists x \exists y P(x, y)) \land (\forall x \forall y \neg Q(x, y)))$

9. (1.6 Rules of Inference) The following argument form is called:

$$p \to q$$
$$q \to r$$
$$\therefore p \to r$$

   a. modus tollens
   b. modus ponens
   c. disjunctive syllogism
   d. hypothetical syllogism

10. (1.6 Rules of Inference) What rule of inference is used in the argument given below?

    If it snows today, the university will close. The university is not closed today.
    Therefore, it did not snow today.

   a. Addition
   b. Simplification
   c. Modus ponens
   d. Modus tollens.

11. (2.1 Sets) Suppose that $A = \{2z, z \in \mathbb{Z}, 0 < z \leq 3\}$, $B = \{2,6\}$, $C = \{4,6\}$, and $D = \{2n, n \in \mathbb{N}, 2 \leq n < 5\}$. Determine which sets are subsets.

   a. $B \subseteq A$
   b. $D \subseteq A$
   c. $A \subseteq D$
   d. $D \subseteq C$

12. (2.2 Set operations) What set does the Venn diagram given below represent?

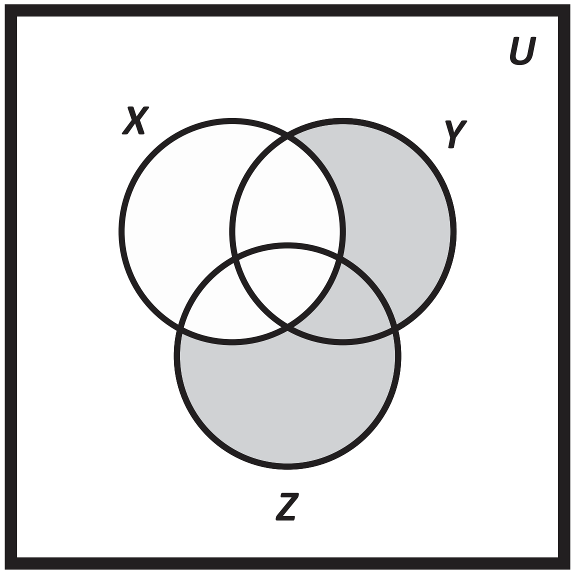


Figure 12. Venn Diagram for Question 12.

a. $(Y - X) \cup (Z - X)$
b. $(X \cap Y) - (X \cap Z)$
c. $(X \cup Y) \cap (X \cup Z)$
d. $(X \cap Y) \cup (X \cap Z)$

13. (2.2 Set operations) Let X, Y and Z be sets. Which of the following is an example of the law of commutativity?

a. $(X \cup Y) \cup Z = X \cup (Y \cup Z)$
b. $(X \cap Y) - Z = Z - (X \cap Y)$
c. $(X \cap Y) \cup Z = (Y \cap X) \cup Z$
d. $X \cap (Y \cup Z) = (X \cup Y) \cap Z$

14. (2.2 Set operations) Which of the following sets is described by the Venn diagram shown below:

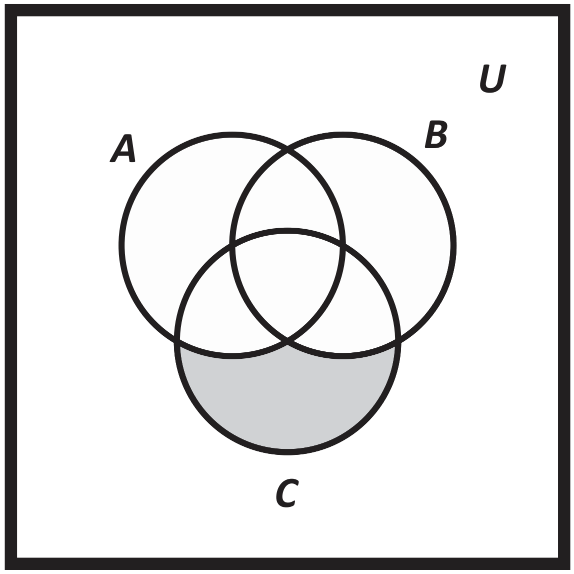


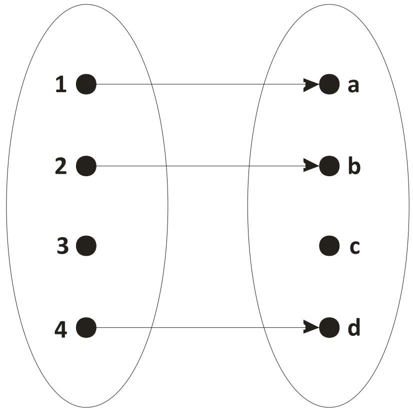
Figure 14. Venn Diagram for Question 14.

a. $\overline{C} \cup \overline{(A \cup B)}$
b. $C \cap \overline{(A \cup B)}$
c. $\left(C \cap \overline{A}\right) \cup (C \cap \overline{B})$
d. $C \cup \overline{(A \cup B)}$

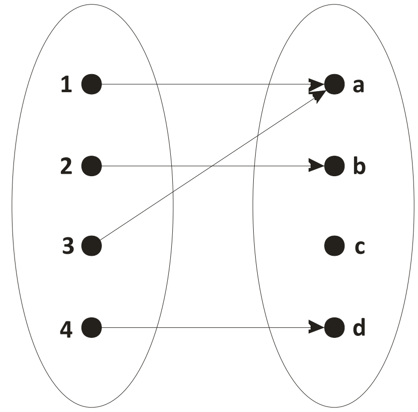
15. (2.3 Functions) Which of the following functions does NOT have an inverse?
a. $X = \{x \in \mathbb{Z} | -2 < x \leq 5\}$, $Y = \{y \in \mathbb{Z} | 1 < y \leq 8\}$, $f: X \to Y$ defined by $f(x) = x + 3$ for all $x \in X$.
b. $f: \mathbb{Z} \to \mathbb{Z}, f(x) = 4x + 7$ for all $x \in \mathbb{Z}$
c. $f: \mathbb{R} \to \mathbb{R}, f(x) = 2x - 5$ for all $x \in \mathbb{R}$.
d. $f: \mathbb{Q} \to \mathbb{Q}, f(x) = 8x$ for all $x \in \mathbb{Q}$.

16. (2.3 Functions) Which of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ is NOT one-to-one?
a. $f(n) = n - 1$
b. $f(n) = n^3$
c. $f(n) = n^2 + 1$
d. $f(n) = -3n + 4$

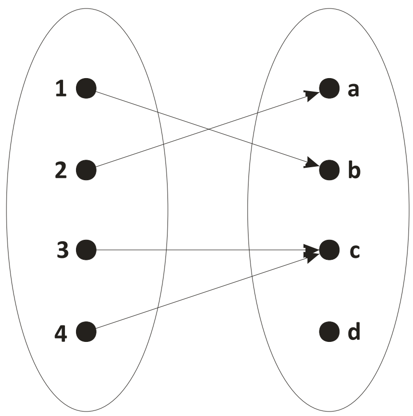
17. (2.3 Functions) Let $A = \{1,2,3,4\}$ and $B = \{a, b, c, d\}$ be sets. Which of the following arrow diagrams is NOT a function from A to B?



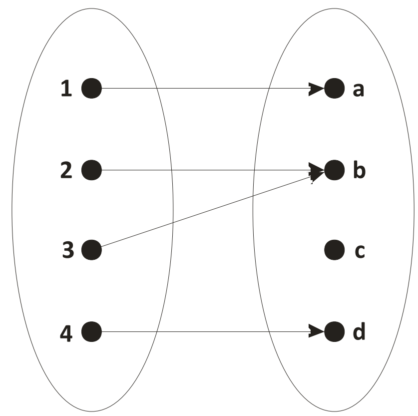
a.



b.



c.



d.

18. (2.4 Sequences and Summations) Select the formula that is used to produce the integer sequence

$$12, 29,70,147,272,457,714, \cdots$$

a. $a_n = 2 + 14n^3, n = 0,1,2,3, \cdots$
b. $a_n = 2n^3, n = 0,1,2,3, \cdots$
c. $a_n = 2n^3 + 3n + 7, n = 1,2,3, \cdots$
d. $a_n = 4 + 14(n^3 - 1) - 2, n = 1,2,3, \cdots$

19. (2.4 Sequences and Summations) Evaluate

$$\sum_{i=0}^{10} \sum_{j=0}^{3} (3i + 2j)$$

a. 627
b. 1025
c. 792
d. 1125

20. (2.4 Sequences and Summations) The first ten integers of a sequence are given below. Find the sixteenth integer in the sequence.

    7, 19, 27, 51, 63, 75, 87, 131, 147, 163, …

    a. 211
    b. 227
    c. 323
    d. 343

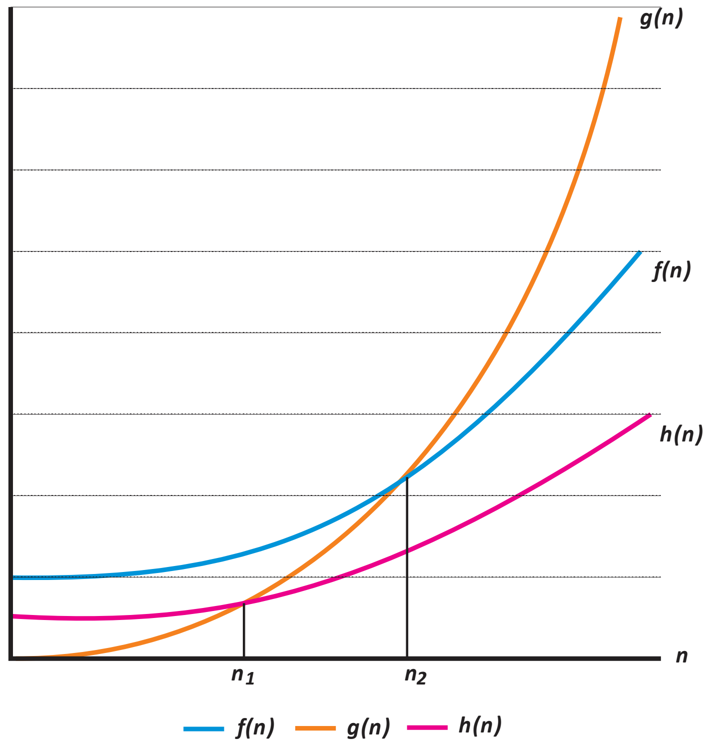
21. (2.4 Sequences and Summations) Find the solution to the recurrence relation and initial condition $a_n = 3a_{n-1}, a_0 = 2$

    a. $a_n = (-3)^n$
    b. $a_n = 2^n \cdot 3$
    c. $a_n = 3^n \cdot 2$
    d. $a_n = 2 \cdot (-3)^n$

22. (Lecture 19 Time Complexity Examples) What is the time complexity of the binary search algorithm?

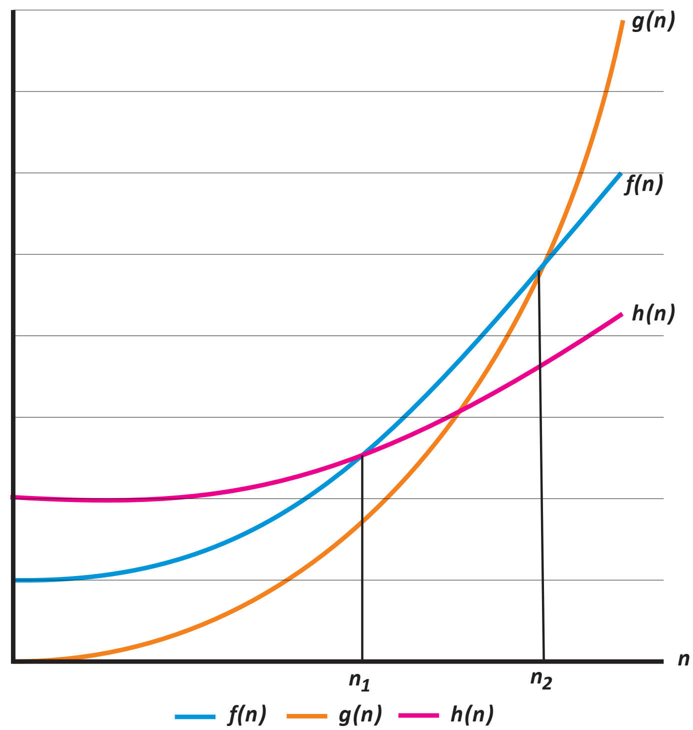
    a. $O(\log n)$
    b. $O(1)$
    c. $O(n)$
    d. $O(n^2)$

23. (Lectures 14 - 19) Select a valid relationship between functions $f(n)$, $g(n)$ and $h(n)$ in the diagram below?



a. $g(n)$ is $O(f(n))$ at n_1
b. $f(n)$ is $O(g(n))$ at n_2
c. $h(n)$ is $\Omega(g(n))$ at n_1
d. $h(n)$ is $\Omega\big(f(n)\big)$ at n_2

24. (Lectures 14 - 19) Select a valid relationship between functions $f(n), g(n)$ and $h(n)$ in the diagram below?



a. $f(n)$ is $O(g(n))$ at n_1
b. $h(n)$ is $O(f(n))$ at n_2
c. $f(n)$ is $\Omega(h(n))$ at n_1
d. $g(n)$ is $\Omega(h(n))$ at n_2

25. (Lecture 15. $T(n)$,Examples) Find the time complexity function $f(n)$ for the code fragment in the Figure 24.1 below. Feel free to employ the table in Figure 24.2 to assist you in your computation of function $f(n)$.

```
int sum=0;
for(int i=0;i<n;i++) {
    for(int j=0;j<i*i;j++) {
        sum++;
    }
}
```

Figure 24.1 Code fragment for Question 24

Line	Code	Cost
1	**int** *sum*=0;	
2	**int** *i*=0;	
3	**while** (*i*<*n*) {	
4	**int** *j*=0;	
5	**while** (*j*<*i***i*) {	
6	*sum*++;	
7	*j*++;	
8	}	
9	*i*++;	
10	}	
	Total	$f(n) =$
	Total	

Figure 24.2 Expanded Code fragment for Question 24

a. $f(n) = 3n^2 + 4n + 3$
b. $f(n) = \frac{3}{2}n^2 + \frac{11}{2}n + 3$
c. $f(n) = \frac{4}{3}n^3 - 2n^2 + \frac{14}{3}n + 3$
d. $f(n) = n^3 + \frac{3}{2}n^2 + \frac{25}{6}n + 3$

Questions 26, 27, and 28 relate to the timing function $f(x) = 3n\lfloor \log_2 n \rfloor + 4n + 3$ of an algorithm.

26. (Lecture 18. $O(n), \Omega(n), \Theta(n)$) Find the best function $g(x)$such that $f(x)$ is $O(g(x))$.

   a. $g(x) = n$
   b. $g(x) = n^2$
   c. $g(x) = 2^n$
   d. $g(x) = n\lfloor \log_2 n \rfloor$

27. (Lecture 18. $O(n), \Omega(n), \Theta(n)$) Find the best value for constant C in the definition of Big-O notation $f(x)$ is $O(g(x))$ where $g(x)$ was determined in the previous question.

   a. $C = 3$
   b. $C = \lim_{n\to\infty} \frac{f(x)}{g(x)}$
   c. $C = 4$
   d. $C = \lim_{n\to\infty} \frac{3\lfloor n \log_2 n \rfloor + 4n + 3}{n^2}$

28. (Lecture 18. $O(n), \Omega(n), \Theta(n)$) Find the smallest integer value for constant k in the definition of Big-O notation $f(x)$ is $O(g(x))$ where $g(x)$ and constant C were determined in the previous questions.

   a. $k = 3$
   b. $k = 18$
   c. $k = 32$
   d. $k = 4$

29. (4.1 Divisibility and Modular Arithmetic) Suppose that a and b are integers, $a \equiv 11 (\mathrm{mod}\ 19)$, and $b \equiv 3 (\mathrm{mod}\ 19)$. Find the integer c with $0 \le c \le 18$ such that $c \equiv a - b (\mathrm{mod}\ 19)$.

   a. $c = 10$
   b. $c = 8$
   c. $c = 3$
   d. $c = 1$

30. (4.1 Divisibility and Modular Arithmetic) What are the quotient and remainder when -111 is divided by 11?

   a. $q = 10, r = 1$
   b. $q = -11, r = 10$
   c. $q = -10, r = -1$
   d. $q = -10, r = -0.09$

31. (4.4 Solving Congruences) Find an inverse of 144 modulo 233.

    a. 7 modulo 233
    b. 55 modulo 233
    c. 89 modulo 233
    d. 34 modulo 233

32. (4.3 Primes and the Greatest Common Divisor) Find the $\gcd(48,116)$.

    a. 16
    b. 4
    c. 8
    d. 2

33. (4.3 Primes and the Greatest Common Divisor) What is the least common multiple of $2^2 \cdot 3^3 \cdot 5^5$ and $2^5 \cdot 3^3 \cdot 5^2$

    a. $2^2 \cdot 3^3 \cdot 5^2$
    b. $2 \cdot 3 \cdot 5$
    c. $2^5 \cdot 3^3 \cdot 5^5$
    d. $2^5 \cdot 3^5 \cdot 5^5$

34. (4.2 Integer Representations and Algorithms) Convert the decimal number 759 to a 12-bit binary number.

    a. $0010\ 1111\ 0111$
    b. $0010\ 1111\ 0101$
    c. $0001\ 1011\ 0111$
    d. $0001\ 1101\ 1111$

35. (4.2 Integer Representations and Algorithms) Compute the binary difference 1100101 - 11011. Both binary numbers are unsigned.

    a. 1001010
    b. 1010100
    c. 1001110
    d. 1101010

36. (4.5 Applications of Congruences) Another way to resolve collisions is to use double hashing. We use an initial hashing function $h(k) = k \textbf{ mod } p$ where p is prime. We also use a second hashing function $g(k) = (k+1)\textbf{mod}(p-2)$. When a collision occurs, we use a *probing sequence* $h(k,i) = (h(k) + i \cdot g(k)) \textbf{ mod } p$.

    Use the double hashing procedure described above with $p = 2029$ to assign memory locations to keys $k_1 = 3534$, $k_2 = 5563$, $k_3 = 3015$, and $k_4 = 5073$. In what memory location is key $k_4$stored?

    a. 1505
    b. 1015
    c. 986
    d. 1975

37. (5.1 Mathematical Induction) Use induction to prove that $P(n)$: $\sum_{i=1}^{n}(2i-1) = n^2 \; \forall \; n \geq 1$. What is the first step in proving $P(n)$ true?

    a. Assume that $P(2), P(3), \cdots, P(k)$ is true.
    b. Prove $\sum_{i=1}^{1}(2 \times i - 1) = 1^2$
    c. Prove $\sum_{i=1}^{1}(2i-1) = (k+1)^2$
    d. Prove $P(k+1)$ is true

38. (5.1 Mathematical Induction) Use induction to prove that $P(n)$: $\sum_{i=1}^{n}(2i-1) = n^2 \; \forall \; n \geq 1$. What is the name of the first step in proving $P(n)$ true?

    a. Inductive hypothesis
    b. Inductive step
    c. Deductive hypothesis
    d. Basis step.

39. (5.1 Mathematical Induction) Use induction to prove that $P(n)$: $\sum_{i=1}^{n}(2i-1) = n^2 \; \forall \; n \geq 1$. What is the inductive step in proving $P(n)$ true?

    a. Prove $\sum_{i=1}^{1}(2 \times i - 1) = 1^2$
    b. Assume that $P(2), P(3), \cdots, P(k)$ is true and prove $P(k+1)$ is true.
    c. Prove $P(k)$ is true
    d. Assume $\sum_{i=1}^{k}(2i-1) = k^2$ and prove $\sum_{i=1}^{k+1}(2(i-1) = (k+1)^2$

40. (5.1 Mathematical Induction) Use induction to prove that $P(n)$: $\sum_{i=1}^{n}(2i-1) = n^2 \; \forall \; n \geq 1$. What is the Inductive Hypothesis in proving $P(n)$?

    a. Prove $\sum_{i=1}^{1}(2 \times i - 1) = 1^2$
    b. Prove $\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$
    c. Assume that $P(k)$ is true.
    d. Prove $P(k+1)$ is true

41. (5.3 Recursive Definition and Structural Induction) Find a formula for $f(n)$ when n is a nonnegative integer given the recursive definition shown below.

$$f(0) = 2, f(1) = 3, f(n) = f(n-1) - 1 \text{ for } n \geq 2$$

   a. $f(n) = 4 - n, n \geq 1$
   b. $f(n) = 2 - n, n \geq 1$
   c. $f(n) = 3 - n, n \geq 1$
   d. $f(n) = 1 - n, n \geq 1$

42. (5.4 Recursive Algorithms) Find a recursive algorithm for finding the sum of the first n positive integers.

int *sum***(int** *n***){return** *n***==1?1:(***n***-1)+***sum***(***n***);}**	**int** *sum***(int** *n***){return** *n***==1?1:***sum***(***n***-1)+***n***;}**
a	**b**
int *sum***(int** *n***){return** *n***==1?1:***sum***(***n***-1)****n***;}**	**int** *sum***(int** *n***){return** *n***==1?1:(***n***-1)****sum***(***n***);}**
c	**d**

43. (6.1 The Basics of Counting) How many bit strings of length eight start with 11 or end with 00?

   a. 64
   b. 112
   c. 16
   d. 128

44. (6.2 The Pigeonhole Principle) What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

   a. 5001
   b. 101
   c. 4951
   d. 51

45. (6.3 Permutations and Combinations) There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

   a. 720
   b. 6
   c. 36
   d. 46,655

46. (6.3 Permutations and Combinations) In how many ways can a set of five letters be selected from the English alphabet?

    a. $P(26{,}5)$
    b. $65{,}780$
    c. $C(5{,}26)$
    d. $\frac{5!}{5!26!}$

47. 6.4 Binomial Coefficients) What is the coefficient of x^5y^8 in $(x+y)^{13}$?

    a. $\binom{13}{5} = \frac{13!}{(13-5)!}$
    b. $\binom{13}{8} = \frac{13!}{(13-8)!}$
    c. $\binom{13}{5} = \frac{13!}{5!(13-5)!}$
    d. $\binom{13}{8} = \frac{13!}{8!(13-8)!}$

48. (6.4 Binomial Coefficients) What is the coefficient of x^5y^{12} in $(3x-2y)^{17}$?

    a. $\frac{17!}{12!5!}3^{12}2^5$
    b. $\frac{17!}{12!5!}3^5 2^{12}$
    c. $-\frac{17!}{12!5!}3^{12}2^5$
    d. $-\frac{17!}{12!5!}3^5 2^{12}$

49. (6.5 Generalized Permutations and Combinations) Every day a student randomly chooses a sandwich for lunch from a pile of wrapped sandwiches. If there are six kinds of sandwiches, how many different ways are there for the student to choose sandwiches for the seven days of a week if the order in which the sandwiches are chosen matters?

    a. $\binom{7}{6}$
    b. 7^6
    c. $\frac{7!}{(7-6)!}$
    d. 6^7

50. (6.5 Generalized Permutations and Combinations) How many ways are there to select five unordered elements from a set with three elements when repetition is allowed?

    a. $\binom{7}{3}$
    b. 3^7
    c. $C(7{,}2)$
    d. $P(7{,}2)$