DEFINITION 1 An algorithm is called *recursive* if it solves a problem by reducing it to an instance of the same problem with smaller input.

EXAMPLE 1 Give a recursive algorithm for computing $n!$, where n is a nonnegative integer.
Solution:

```
unsigned int factorial(unsigned int n) {return n>0?n*factorial(n-1):1;}
```

EXAMPLE 2 Give a recursive algorithm for computing $a^n$where n is a nonnegative integer.
Solution:

```
double power(double b,unsigned int n) {return n>0?b*power(n-1):1;}
```

EXAMPLE 4 Give a recursive algorithm for computing the greatest common divisor of two non-negative integers a and b with $a < b$.
Solution:

```
unsigned int gcd(unsigned int a, unsigned int b){return a==0?b:gcd(b%a,a);}
int main()
{   for (;;) {
        cout << endl;
        cout << "Enter nonnegative integer argument a. ";
        int a;
        cin  >> a;
        cout << "Enter nonnegative integer argument b. ";
        int b;
        cin  >> b;
        if (a<0||b<0) break;
        if (a>=b) {
          cout << endl << a << " must be less than " << b << ".";
          continue;
        }
        cout << "gcd(" << a << "," << b << ")=" << gcd(a,b);
    }
    return 0;
}
```

EXAMPLE 7 Prove that Algorithm 2, which computes powers of real numbers, is correct.

Solution: We use mathematical induction on the exponent n.

BASIS STEP: If $n = 0$, the first step of the algorithm tells us that $power(b, 0) = 1$. This is correct because $b^0 = 1$ for every nonzero real number b. This completes the basis step.

INDUCTIVE STEP: The inductive hypothesis is the statement that $power(b, k) = b^k$ for all $b \neq 0$ for the nonnegative integer k. That is, the inductive hypothesis is the statement that the algorithm correctly computes b^k. To complete the inductive step, we show that if the inductive hypothesis is true, then the algorithm correctly computes b^{k+1}. Because $k + 1$ is a positive integer, when the algorithm computes b^{k+1}, the algorithm sets $power(b, k + 1) = b \cdot power(b, k) = b \cdot b^k = b^{k+1}$. This completes the inductive step.