Chapter 1

Use the following to answer questions 1-5:

In the questions below determine whether the proposition is TRUE or FALSE

1. $1 + 1 = 3$ if and only if $2 + 2 = 3$.
   Ans: True

2. If it is raining, then it is raining.
   Ans: True

3. If $1 < 0$, then $3 = 4$.
   Ans: True

4. If $2 + 1 = 3$, then $2 = 3 - 1$.
   Ans: True

5. If $1 + 1 = 2$ or $1 + 1 = 3$, then $2 + 2 = 3$ and $2 + 2 = 4$.
   Ans: False

6. Write the truth table for the proposition $\neg(r \to \neg q) \vee (p \wedge \neg r)$.

	q	r	$\neg(r \to \neg q) \vee (p \wedge \neg r)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

7. (a) Find a proposition with the given truth table.

p	q	?
T	T	F
T	F	F
F	T	T
F	F	F

(b) Find a proposition using only p,q,$\neg$, and the connective $\vee$ that has this truth table.
   Ans: (a) $\neg p \wedge q$, (b) $\neg(p \vee \neg q)$.

8. Find a proposition with three variables p, q, and r that is true when p and r are true and q is false, and false otherwise
   Ans: (a) $p \wedge \neg q \wedge r$.

9. Find a proposition with three variables p, q, and r that is true when exactly one of the three variables is true, and false otherwise
   Ans: $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$.

10. Find a proposition with three variables p, q, and r that is never true
    Ans: $(p \wedge \neg p) \vee (q \wedge \neg q) \vee (r \wedge \neg r)$.

11. Find a proposition using only p,q,$\neg$ and the connective $\vee$ with the given truth table.

p	q	?
T	T	F
T	F	T
F	T	T
F	F	F

    Ans: $\neg(\neg p \vee q) \vee \neg(p \vee \neg q)$.

12. Determine whether $p \rightarrow (q \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are equivalent.
    Ans: Not equivalent. Let q be false and p and r be true.

13. Determine whether $p \rightarrow (q \rightarrow r)$ is equivalent to $(p \rightarrow q) \rightarrow r$.
    Ans: Not equivalent. Let p, q, and r be false.

14. Determine whether $(p \rightarrow q) \wedge (\neg p \rightarrow q) \equiv q$.
    Ans: Both truth tables are identical:

p	q	$(p \rightarrow q) \wedge (\neg p \rightarrow q)$	q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	F	F

15. Write a proposition equivalent to $p \vee \neg q$ that uses only p,q,$\neg$ and the connective $\wedge$.
    Ans: $\neg(\neg p \wedge q)$.

16. Write a proposition equivalent to $\neg p \wedge \neg q$ using only p,q,$\neg$ and the connective $\vee$.
    Ans: $\neg(p \vee q)$.

17. Prove that the proposition "if it is not hot, then it is hot" is equivalent to "it is hot".
Ans: Both propositions are true when "it is hot" is true and both are false when "it is hot" is false.

18. Write a proposition equivalent to $p \to q$ using only $p, q, \neg$ and the connective: $\vee$.
Ans: $\neg p \vee q$.

19. Write a proposition equivalent to $p \to q$ using only $p, q, \neg$ and the connective $\wedge$.
Ans: $\neg(p \wedge \neg q)$.

20. Prove that $p \to q$ and its converse are not logically equivalent.
Ans: Truth values differ when p is true and q is false.

21. Prove that $\neg p \to \neg q$ and its inverse are not logically equivalent.
Ans: Truth values differ when p is false and q is true.

22. Determine whether the following two propositions are logically equivalent: $p \vee (q \wedge r)$, $(p \wedge q) \vee (p \wedge r)$.
Ans: No.

23. Determine whether the following two propositions are logically equivalent: $p \to (\neg q \wedge r)$, $\neg p \vee \neg(r \to q)$.
Ans: Yes.

24. Prove that $(q \wedge (p \to \neg q)) \to \neg p$ is a tautology using propositional equivalence and the laws of logic.

Ans:

$$
\begin{aligned}
& (q \wedge (p \to \neg q)) \to \neg p \\
& \Longleftrightarrow (q \wedge (\neg p \vee \neg q)) \to \neg p \\
& \Longleftrightarrow ((q \wedge \neg p) \vee (q \wedge \neg q)) \to \neg p \\
& \Longleftrightarrow (q \wedge \neg p) \to \neg p \\
& \Longleftrightarrow \neg(q \wedge \neg p) \vee \neg p \\
& \Longleftrightarrow (\neg q \vee p) \vee \neg p \\
& \Longleftrightarrow \neg q \vee (p \vee \neg p)
\end{aligned}
$$

, which is always true.

25. Determine whether this proposition is a tautology: $((p \to q) \wedge \neg p) \to \neg q$.
Ans: No.

26. Determine whether this proposition is a tautology: $((p \to \neg q) \wedge q) \to \neg p$.
Ans: Yes.

Use the following to answer questions 27-33:

In the questions below write the statement in the form "If ..., then ...."

27. *x* is even only if *y* is odd.
    Ans: If *x* is even, then *y* is odd.

28. *A* implies *B*.
    Ans: If *A*, then *B*.

29. It is hot whenever it is sunny.
    Ans: If it is sunny, then it is hot.

30. To get a good grade it is necessary that you study.
    Ans: If you don't study, then you don't get a good grade (equivalently, if you get a good grade, then you study).

31. Studying is sufficient for passing.
    Ans: If you study, then you pass.

32. The team wins if the quarterback can pass.
    Ans: If the quarterback can pass, then the team wins.

33. You need to be registered in order to check out library books.
    Ans: If you are not registered, then you cannot check out library books (equivalently, if you check out library books, then you are registered).

34. Write the contrapositive, converse, and inverse of the following: If you try hard, then you will win.
    Ans: Contrapositive: If you will not win, then you do not try hard. Converse: If you will win, then you try hard. Inverse: If you do not try hard, then you will not win.

35. Write the contrapositive, converse, and inverse of the following: You sleep late if it is Saturday.
    Ans: Contrapositive: If you do not sleep late, then it is not Saturday. Converse: If you sleep late, then it is Saturday. Inverse: If it is not Saturday, then you do not sleep late.

Use the following to answer questions 36-38:

In the questions below write the negation of the statement. (Don't write "It is not true that ....")

36. It is Thursday and it is cold.
    Ans: It is not Thursday or it is not cold.

37. I will go to the play or read a book, but not both.
   Ans: I will go to the play and read a book, or I will not go to the play and not read a book.

38. If it is rainy, then we go to the movies.
   Ans: It is rainy and we do not go to the movies.

39. Explain why the negation of “Al and Bill are absent” is not “Al and Bill are present”.
   Ans: Both propositions can be false at the same time. For example, Al could be present and Bill absent.

40. Using c for “it is cold” and d for “it is dry”, write “It is neither cold nor dry” in symbols.
   Ans: $\neg c \wedge \neg d$.

41. Using c for “it is cold” and r for “it is rainy”, write “It is rainy if it is not cold” in symbols.
   Ans: $\neg c \rightarrow r$.

42. Using c for “it is cold” and w for “it is windy”, write “To be windy it is necessary that it be cold” in symbols.
   Ans: $w \rightarrow c$.

43. Using c for “it is cold”, r for “it is rainy”, and w for “it is windy”, write “It is rainy only if it is windy and cold” in symbols.
   Ans: $r \rightarrow (w \wedge c)$.

44. A set of propositions is *consistent* if there is an assignment of truth values to each of the variables in the propositions that makes each proposition true. Is the following set of propositions consistent?
   The system is in multiuser state if and only if it is operating normally.
   If the system is operating normally, the kernel is functioning.
   The kernel is not functioning or the system is in interrupt mode.
   If the system is not in multiuser state, then it is in interrupt mode.
   The system is in interrupt mode.
   Ans: Using m, n, k, and i, there are three rows of the truth table that have all five propositions true: the rows TTTT, FFTT, FFFT for m,n,k,i.

45. On the island of knights and knaves you encounter two people, A and B. Person A says, "B is a knave." Person B says, "We are both knights." Determine whether each person is a knight or a knave.
   Ans: A is a knight, B is a knave.

46. On the island of knights and knaves you encounter two people. *A* and *B*. Person *A* says, "*B* is a knave." Person *B* says, "At least one of us is a knight." Determine whether each person is a knight or a knave.
    Ans: *A* is a knave, *B* is a knight.

Use the following to answer questions 47-49:

In the questions below suppose that $Q(x)$ is "$x + 1 = 2x$", where x is a real number. Find the truth value of the statement.

47. $Q(2)$.
    Ans: False

48. $\forall x Q(x)$.
    Ans: False

49. $\exists x Q(x)$.
    Ans: True

Use the following to answer questions 50-57:

In the questions below $P(x,y)$ means "$x + 2y = xy$", where x and y are integers. Determine the truth value of the statement.

50. $P(1,-1)$.
    Ans: True

51. $P(0,0)$.
    Ans: True

52. $\exists y P(3,y)$.
    Ans: True

53. $\forall x \exists y P(x,y)$.
    Ans: False

54. $\exists x \forall y P(x,y)$.
    Ans: False

55. $\forall y \exists x P(x,y)$.
    Ans: False

56. $\exists y \forall x P(x,y)$.
    Ans: False

57. $\neg\forall x\exists y\neg P(x,y)$.
Ans: False

Use the following to answer questions 58-59:

In the questions below $P(x,y)$ means "x and y are real numbers such that $x + 2y = 5$". Determine whether the statement is true.

58. $\forall x\exists yP(x,y)$.
Ans: True, since for every real number x we can find a real number y such that $x + 2y = 5$, namely $y = (5 - x)/2$.

59. $\exists x\forall yP(x,y)$.
Ans: False, if it were true for some number x_0, then $x_0 = 5 - 2y$ for every y, which is not possible.

Use the following to answer questions 60-62:

In the questions below $P(m,n)$ means "$m \le n$", where the universe of discourse for m and n is the set of nonnegative integers. What is the truth value of the statement?

60. $\forall nP(0,n)$.
Ans: True

61. $\exists n\forall mP(m,n)$.
Ans: False

62. $\forall m\exists nP(m,n)$.
Ans: True

Use the following to answer questions 63-68:

In the questions below suppose $P(x,y)$ is a predicate and the universe for the variables x and y is $\{1,2,3\}$. Suppose $P(1,3)$, $P(2,1)$, $P(2,2)$, $P(2,3)$, $P(2,3)$, $P(3,1)$, $P(3,2)$ are true, and $P(x,y)$ is false otherwise. Determine whether the following statements are true.

63. $\forall x\exists yP(x,y)$.
Ans: True

64. $\exists x\forall yP(x,y)$.
Ans: True

65. $\neg\exists x\exists y(P(x,y) \wedge \neg P(y,x))$.
Ans: False

66. $\forall y \exists x(P(x,y) \rightarrow P(y,x))$.
Ans: True

67. $\forall x \forall y\ (x \neq y \rightarrow (P(x,y) \vee P(y,x))$.
Ans: False

68. $\forall y \exists x\ (x \leq y \wedge (P(x,y))$.
Ans: False

Use the following to answer questions 69-72:

In the questions below suppose the variable x represents students and y represents courses, and:
$U(y)$: y is an upper-level course $M(y)$: y is a math course $F(x)$: x is a freshman
$B(x)$: x is a full-time student $T(x,y)$: student x is taking course y.
Write the statement using these predicates and any needed quantifiers.

69. Eric is taking MTH 281.
Ans: T(Eric, MTH 281).

70. All students are freshmen.
Ans: $\forall x F(x)$.

71. Every freshman is a full-time student.
Ans: $\forall x(F(x) \rightarrow B(x))$.

72. No math course is upper-level.
Ans: $\forall y(M(y) \rightarrow \neg U(y))$.

Use the following to answer questions 73-75:

In the questions below suppose the variable x represents students and y represents courses, and:
$U(y)$: y is an upper-level course $M(y)$: y is a math course $F(x)$: x is a freshman
$A(x)$: x is a part-time student $T(x,y)$: student x is taking course y.
Write the statement using these predicates and any needed quantifiers.

73. Every student is taking at least one course.
Ans: $\forall x \exists y\ T(x,y)$.

74. There is a part-time student who is not taking any math course.
Ans: $\exists x \forall y[A(x) \wedge (M(y) \rightarrow \neg T(x,y))]$.

75. Every part-time freshman is taking some upper-level course.
Ans: $\forall x \exists y[(F(x) \wedge A(x)) \rightarrow (U(y) \wedge T(x,y))]$.

Use the following to answer questions 76-78:

In the questions below suppose the variable x represents students and y represents courses, and:
$F(x)$: x is a freshman $A(x)$: x is a part-time student $T(x,y)$: x is taking y.
Write the statement in good English without using variables in your answers.

76. $F(\text{Mikko})$.
Ans: Mikko is a freshman.

77. $\neg\exists y T(\text{Joe},y)$.
Ans: Joe is not taking any course.

78. $\exists x(A(x) \wedge \neg F(x))$.
Ans: Some part-time students are not freshmen.

Use the following to answer questions 79-81:

In the questions below suppose the variable x represents students and y represents courses, and:
$M(y)$: y is a math course $F(x)$: x is a freshman
$B(x)$: x is a full-time student $T(x,y)$: x is taking y.
Write the statement in good English without using variables in your answers.

79. $\forall x\exists y T(x,y)$.
Ans: Every student is taking a course.

80. $\exists x\forall y T(x,y)$.
Ans: Some student is taking every course.

81. $\forall x\exists y[(B(x) \wedge F(x)) \rightarrow (M(y) \wedge T(x,y))]$.
Ans: Every full-time freshman is taking a math course.

Use the following to answer questions 82-84:

In the questions below suppose the variables x and y represent real numbers, and
$L(x,y) : x < y$ $G(x) : x > 0$ $P(x)$: x is a prime number.
Write the statement in good English without using any variables in your answer.

82. $L(7,3)$.
Ans: $7 < 3$.

83. $\forall x\exists y L(x,y)$.
Ans: There is no largest number.

84. $\forall x\exists y[G(x) \rightarrow (P(y) \wedge L(x,y))]$.
Ans: No matter what positive number is chosen, there is a larger prime.

Use the following to answer questions 85-87:

In the questions below suppose the variables x and y represent real numbers, and
$L(x,y) : x < y \quad Q(x,y) : x = y \quad E(x) : x$ is even $\quad I(x) : x$ is an integer.
Write the statement using these predicates and any needed quantifiers.

85. Every integer is even.
Ans: $\forall x(I(x) \rightarrow E(x))$.

86. If $x < y$, then x is not equal to y.
Ans: $\forall x \forall y(L(x,y) \rightarrow \neg Q(x,y))$.

87. There is no largest real number.
Ans: $\forall x \exists y L(x,y)$.

Use the following to answer questions 88-89:

In the questions below suppose the variables x and y represent real numbers, and
$E(x) : x$ is even $\quad G(x) : x > 0 \quad I(x) : x$ is an integer.
Write the statement using these predicates and any needed quantifiers.

88. Some real numbers are not positive.
Ans: $\exists x \neg G(x)$.

89. No even integers are odd.
Ans: $\neg\exists x(I(x) \wedge E(x) \wedge \neg E(x)])$.

Use the following to answer questions 90-92:

In the questions below suppose the variable x represents people, and
$F(x)$: x is friendly $\;T(x)$: x is tall $\;A(x)$: x is angry.
Write the statement using these predicates and any needed quantifiers.

90. Some people are not angry.
Ans: $\exists x \neg A(x)$.

91. All tall people are friendly.
Ans: $\forall x(T(x) \rightarrow F(x))$.

92. No friendly people are angry.
Ans: $\forall x(F(x) \rightarrow \neg A(x))$.

Use the following to answer questions 93-94:

In the questions below suppose the variable x represents people, and

$F(x)$: x is friendly $T(x)$: x is tall $A(x)$: x is angry.

Write the statement using these predicates and any needed quantifiers.

93. Some tall angry people are friendly.
    Ans: $\exists x(T(x) \wedge A(x) \wedge F(x))$.

94. If a person is friendly, then that person is not angry.
    Ans: $\forall x(F(x) \rightarrow \neg A(x))$.

Use the following to answer questions 95-97:

In the questions below suppose the variable x represents people, and

$F(x)$: x is friendly $T(x)$: x is tall $A(x)$: x is angry.

Write the statement in good English. Do not use variables in your answer.

95. $A(\text{Bill})$.
    Ans: Bill is angry.

96. $\neg\exists x(A(x) \wedge T(x))$.
    Ans: No one is tall and angry.

97. $\neg\forall x(F(x) \rightarrow A(x))$.
    Ans: Some friendly people are not angry.

Use the following to answer questions 98-100:

In the questions below suppose the variable x represents students and the variable y represents courses, and

$A(y)$: y is an advanced course $S(x)$: x is a sophomore

$F(x)$: x is a freshman $T(x,y)$: x is taking y.

Write the statement using these predicates and any needed quantifiers.

98. There is a course that every freshman is taking.
    Ans: $\exists y\forall x(F(x) \rightarrow T(x,y))$.

99. No freshman is a sophomore.
    Ans: $\neg\exists x(F(x) \wedge S(x)]$.

100. Some freshman is taking an advanced course.
    Ans: $\exists x\exists y(F(x) \wedge A(y) \wedge T(x,y))$.

Use the following to answer questions 101-102:

In the questions below suppose the variable x represents students and the variable y represents courses, and

$A(y)$: y is an advanced course $F(x)$: x is a freshman
$T(x,y)$: x is taking y $P(x,y)$: x passed y.

Write the statement using the above predicates and any needed quantifiers.

101. No one is taking every advanced course.
Ans: $\neg\exists x\forall y\,(A(y) \rightarrow T(x,y))$.

102. Every freshman passed calculus.
Ans: $\forall x(F(x) \rightarrow P(x,calculus))$.

Use the following to answer questions 103-105:

In the questions below suppose the variable x represents students and the variable y represents courses, and

$T(x,y)$: x is taking y $P(x,y)$: x passed y.

Write the statement in good English. Do not use variables in your answers.

103. $\neg P$(Wisteria,MAT100).
Ans: Wisteria did not pass MAT 100.

104. $\exists y\forall x T(x,y)$.
Ans: There is a course that all students are taking.

105. $\forall x\exists y T(x,y)$.
Ans: Every student is taking at least one course.

Use the following to answer questions 106-110:

In the questions below assume that the universe for x is all people and the universe for y is the set of all movies. Write the English statement using the following predicates and any needed quantifiers:

$S(x,y)$: x saw y $L(x,y)$: x liked y $A(y)$: y won an award $C(y)$: y is a comedy.

106. No comedy won an award.
Ans: $\forall y[C(y) \rightarrow \neg A(y)]$.

107. Lois saw *Casablanca*, but didn't like it.
Ans: S(Lois,*Casablanca*) $\wedge$ $\neg L$(Lois,*Casablanca*).

108. Some people have seen every comedy.
Ans: $\exists x\forall y\,[C(y) \rightarrow S(x,y)]$.

109. No one liked every movie he has seen.
Ans: $\neg\exists x\forall y[S(x,y) \rightarrow L(x,y)]$.

110. Ben has never seen a movie that won an award.
Ans: $\neg\exists y[A(y) \wedge S(\text{Ben},y)]$.

Use the following to answer questions 111-113:

In the questions below assume that the universe for x is all people and the universe for y is the set of all movies. Write the statement in good English, using the predicates

$S(x,y)$: x saw y $L(x,y)$: x liked y.

Do not use variables in your answer.

111. $\exists y\neg S(\text{Margaret},y)$.
Ans: There is a movie that Margaret did not see.

112. $\exists y\forall xL(x,y)$.
Ans: There is a movie that everyone liked.

113. $\forall x\exists yL(x,y)$.
Ans: Everyone liked at least one movie.

Use the following to answer questions 114-123:

In the questions below suppose the variable x represents students, y represents courses, and $T(x,y)$ means "x is taking y". Match the English statement with all its equivalent symbolic statements in this list:

1. $\exists x\forall yT(x,y)$
2. $\exists y\forall xT(x,y)$
3. $\forall x\exists yT(x,y)$
4. $\neg\exists x\exists yT(x,y)$
5. $\exists x\forall y\neg T(x,y)$
6. $\forall y\exists xT(x,y)$
7. $\exists y\forall x\neg T(x,y)$
8. $\neg\forall x\exists yT(x,y)$
9. $\neg\exists y\forall xT(x,y)$
10. $\neg\forall x\exists y\neg T(x,y)$
11. $\neg\forall x\neg\forall y\neg T(x,y)$
12. $\forall x\exists y\neg T(x,y)$

114. Every course is being taken by at least one student.
Ans: 6.

115. Some student is taking every course.
Ans: 1, 10.

116. No student is taking all courses.
Ans: 12.

117. There is a course that all students are taking.
Ans: 2.

118. Every student is taking at least one course.
Ans: 3.

119. There is a course that no students are taking.
Ans: 7.

120. Some students are taking no courses.
Ans: 5, 8, 11.

121. No course is being taken by all students.
Ans: 9.

122. Some courses are being taken by no students.
Ans: 7.

123. No student is taking any course.
Ans: 4.

Use the following to answer questions 124-134:

In the questions below suppose the variable x represents students, $F(x)$ means "x is a freshman", and $M(x)$ means "x is a math major". Match the statement in symbols with one of the English statements in this list:

1. Some freshmen are math majors.
2. Every math major is a freshman.
3. No math major is a freshman.

124. $\forall x(M(x) \rightarrow \neg F(x))$.
Ans: 3.

125. $\neg\exists x(M(x) \wedge \neg F(x))$.
Ans: 2.

126. $\forall x(F(x) \rightarrow \neg M(x))$.
Ans: 3.

127. $\forall x(M(x) \rightarrow F(x))$.
Ans: 2.

128. $\exists x(F(x) \wedge M(x))$.
Ans: 1.

129. $\neg\forall x(\neg F(x) \vee \neg M(x))$.
Ans: 1.

130. $\forall x(\neg(M(x) \wedge \neg F(x)))$.
Ans: 2.

131. $\forall x(\neg M(x) \vee \neg F(x))$.
Ans: 3.

132. $\neg\exists x(M(x) \wedge \neg F(x))$.
Ans: 2.

133. $\neg\exists x(M(x) \wedge F(x))$.
Ans: 3.

134. $\neg\forall x(F(x) \rightarrow \neg M(x))$.
Ans: 1.

Use the following to answer questions 135-138:

In the questions below let $F(A)$ be the predicate "A is a finite set" and $S(A,B)$ be the predicate "A is contained in B". Suppose the universe of discourse consists of all sets. Translate the statement into symbols.

135. Not all sets are finite.
Ans: $\exists A \neg F(A)$.

136. Every subset of a finite set is finite.
Ans: $\forall A \forall B[(F(B) \wedge S(A,B)) \rightarrow F(A)]$.

137. No infinite set is contained in a finite set.
Ans: $\neg\exists A \exists B(\neg F(A) \wedge F(B) \wedge S(A,B))$.

138. The empty set is a subset of every finite set.
Ans: $\forall A[F(A) \rightarrow S(\varnothing,A)]$.

Use the following to answer questions 139-143:

In the questions below write the negation of the statement in good English. Don't write "It is not true that ...."

139. Some bananas are yellow.
Ans: No bananas are yellow.

140. All integers ending in the digit 7 are odd.
Ans: Some integers ending in the digit 7 are not odd.

141. No tests are easy.
Ans: Some tests are easy.

142. Roses are red and violets are blue.
Ans: Roses are not red or violets are not blue.

143. Some skiers do not speak Swedish.
Ans: All skiers speak Swedish.

144. A student is asked to give the negation of "all bananas are ripe".
(a) The student responds "all bananas are not ripe". Explain why the English in the student's response is ambiguous.
(b) Another student says that the negation of the statement is "no bananas are ripe". Explain why this is not correct.
(c) Another student says that the negation of the statement is "some bananas are ripe". Explain why this is not correct.
(d) Give the correct negation.
Ans: (a) Depending on which word is emphasized, the sentence can be interpreted as "all bananas are non-ripe fruit" (i.e., no bananas are ripe) or as "not all bananas are ripe" (i.e., some bananas are not ripe).
(b) Both statements can be false at the same time.
(c) Both statements can be true at the same time.
(d) Some bananas are not ripe.

145. Explain why the negation of "Some students in my class use e-mail" is not "Some students in my class do not use e-mail".
Ans: Both statements can be true at the same time.

146. What is the rule of inference used in the following:
If it snows today, the university will be closed. The university will not be closed today. Therefore, it did not snow today.
Ans: Modus tollens.

147. What is the rule of inference used in the following:
If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.
Ans: Hypothetical syllogism.

148. Explain why an argument of the following form is not valid:

$p \to q$
$\neg p$
____
$\therefore \neg q$

Ans: p false and q true yield true hypotheses but a false conclusion.

149. Determine whether the following argument is valid:

$p \to r$
$q \to r$
$\neg(p \vee q)$
____
$\therefore \neg r$

Ans: Not valid: p false, q false, r true

150. Determine whether the following argument is valid:

$p \to r$
$q \to r$
$q \vee \neg r$
____
$\therefore \neg p$

Ans: Not valid: p true, q true, r true

151. Show that the hypotheses "I left my notes in the library or I finished the rough draft of the paper" and "I did not leave my notes in the library or I revised the bibliography" imply that "I finished the rough draft of the paper or I revised the bibliography".
Ans: Use resolution on $l \vee f$ and $\neg l \vee r$ to conclude $f \vee r$.

152. Determine whether the following argument is valid. Name the rule of inference or the fallacy.
If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.
Ans: Not valid: fallacy of affirming the conclusion.

153. Determine whether the following argument is valid. Name the rule of inference or the fallacy.
If n is a real number such that $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.
Ans: Not valid: fallacy of denying the hypothesis.

154. Determine whether the following argument is valid:
She is a Math Major or a Computer Science Major.
If she does not know discrete math, she is not a Math Major.
If she knows discrete math, she is smart.
She is not a Computer Science Major.
Therefore, she is smart.
Ans: Valid.

155. Determine whether the following argument is valid.
Rainy days make gardens grow.
Gardens don't grow if it is not hot.
It always rains on a day that is not hot.
Therefore, if it is not hot, then it is hot.
Ans: Valid.

156. Determine whether the following argument is valid.
If you are not in the tennis tournament, you will not meet Ed.
If you aren't in the tennis tournament or if you aren't in the play, you won't meet Kelly.
You meet Kelly or you don't meet Ed.
It is false that you are in the tennis tournament and in the play.
Therefore, you are in the tennis tournament.
Ans: Not valid.

157. Show that the premises "Every student in this class passed the first exam" and "Alvina is a student in this class" imply the conclusion "Alvina passed the first exam".
Ans: Universal instantiation.

158. Show that the premises "Jean is a student in my class" and "No student in my class is from England" imply the conclusion "Jean is not from England".
Ans: Universal instantiation.

159. Determine whether the premises "Some math majors left the campus for the weekend" and "All seniors left the campus for the weekend" imply the conclusion "Some seniors are math majors".
Ans: The two premises do not imply the conclusion.

160. Show the premises "Everyone who read the textbook passed the exam", and "Ed read the textbook" imply the conclusion "Ed passed the exam".
Ans: Let R(x) be the predicate "x has read the textbook" and P(x) be the predicate "x passed the exam". The following is the proof:

1. $\forall x(R(x) \to P(x))$	hypothesis
2. $R(\text{Ed}) \to P(\text{Ed})$	universal instantiation on 1
3. $R(\text{Ed})$	hypothesis
4. $P(\text{Ed})$	modus ponens on 2 and 3

161. Determine whether the premises "No juniors left campus for the weekend" and "Some math majors are not juniors" imply the conclusion "Some math majors left campus for the weekend."
Ans: The two premises do not imply the conclusion.

162. Show that the premise “My daughter visited Europe last week” implies the conclusion “Someone visited Europe last week”.
Ans: Existential generalization.

163. Suppose you wish to prove a theorem of the form “if p then q”.
(a) If you give a direct proof, what do you assume and what do you prove?
(b) If you give an indirect proof, what do you assume and what do you prove?
(c) If you give a proof by contradiction, what do you assume and what do you prove?
Ans: (a) Assume p, prove q.
(b) Assume $\neg q$, prove $\neg p$.
(c) Assume $p \wedge \neg q$, show that this leads to a contradiction.

164. Suppose that you had to prove a theorem of the form “if p then q”. Explain the difference between a direct proof and a proof by contraposition.
Ans: Direct proof: Assume p, show q. Indirect proof: Assume $\neg q$, show $\neg p$.

165. Give a direct proof of the following: “If x is an odd integer and y is an even integer, then $x + y$ is odd”.
Ans: Suppose $x = 2k + 1$, $y = 2l$. Therefore $x + y = 2k + 1 + 2l = 2(k + l) + 1$, which is odd.

166. Give a proof by contradiction of the following: “If n is an odd integer, then n^2 is odd”.
Ans: Suppose $n = 2k + 1$ but $n^2 = 2l$. Therefore $(2k + 1)^2 = 2l$, or $4k^2 + 4k + 1 = 2l$. Hence $2(2k^2 + 2k - l) = -1$ (even = odd), a contradiction. Therefore n^2 is odd.

167. Consider the following theorem: “if x and y are odd integers, then $x + y$ is even”. Give a direct proof of this theorem.
Ans: Let $x = 2k + 1$, $y = 2l + 1$. Therefore $x + y = 2k + 1 + 2l + 1 = 2(k + l + 1)$, which is even.

168. Consider the following theorem: “if x and y are odd integers, then $x + y$ is even”. Give a proof by contradiction of this theorem.
Ans: Suppose $x = 2k + 1$ and $y = 2l + 1$, but $x + y = 2m + 1$. Therefore $(2k + 1) + (2l + 1) = 2m + 1$. Hence $2(k + l - m + 1) = 1$ (even = odd), which is a contradiction. Therefore $x + y$ is even.

169. Give a proof by contradiction of the following: If x and y are even integers, then xy is even.
Ans: Suppose $x = 2k$ and $y = 2l$, but $xy = 2m + 1$. Therefore $2k \cdot 2l = 2m + 1$. Hence $2(2kl - m) = 1$ (even = odd), which is a contradiction. Therefore xy is even.

170. Consider the following theorem: If x is an odd integer, then $x + 2$ is odd. Give a direct proof of this theorem
Ans: Let $x = 2k + 1$. Therefore $x + 2 = 2k + 1 + 2 = 2(k + 1) + 1$, which is odd.

171. Consider the following theorem: If x is an odd integer, then $x + 2$ is odd. Give a proof by contraposition of this theorem.
Ans: Suppose $x + 2 = 2k$. Therefore $x = 2k - 2 = 2(k - 1)$, which is even.

172. Consider the following theorem: If x is an odd integer, then $x + 2$ is odd. Give a proof by contradiction of this theorem.
Ans: Suppose x is odd but $x + 2$ is even. Therefore $x = 2k + 1$ and $x + 2 = 2l$. Hence $(2k + 1) + 2 = 2l$. Therefore $2(k + 1 - l) = -1$ (even = odd), a contradiction.

173. Consider the following theorem: If n is an even integer, then $n + 1$ is odd. Give a direct proof of this theorem.
Ans: Let $n = 2k$. Therefore $n + 1 = 2k + 1$, which is odd.

174. Consider the following theorem: If n is an even integer, then $n + 1$ is odd. Give a proof by contraposition of this theorem.
Ans: Suppose $n + 1$ is even. Therefore $n + 1 = 2k$. Therefore $n = 2k - 1 = 2(k - 1) + 1$, which is odd.

175. Consider the following theorem: If n is an even integer, then $n + 1$ is odd. Give a proof by contradiction of this theorem.
Ans: Suppose $n = 2k$ but $n + 1 = 2l$. Therefore $2k + 1 = 2l$ (even = odd), which is a contradiction.

176. Prove that the following is true for all positive integers n: n is even if and only if $3n^2 + 8$ is even.
Ans: If n is even, then $n = 2k$. Therefore $3n^2 + 8 = 3(2k)^2 + 8 = 12k^2 + 8 = 2(6k^2 + 4)$, which is even. If n is odd, then $n = 2k + 1$. Therefore $3n^2 + 8 = 3(2k + 1)^2 + 8 = 12k^2 + 12k + 11 = 2(6k^2 + 6k + 5) + 1$, which is odd.

177. Prove the following theorem: n is even if and only if n^2 is even.
Ans: If n is even, then $n^2 = (2k)^2 = 2(2k^2)$, which is even. If n is odd, then $n^2 = (2k + 1)^2 = 2(2k^2 + 2k) + 1$, which is odd.

178. Prove: if m and n are even integers, then mn is a multiple of 4.
Ans: If $m = 2k$ and $n = 2l$, then $mn = 4kl$. Hence mn is a multiple of 4.

179. Prove or disprove: For all real numbers x and y, $\lfloor x - y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$.
Ans: False: $x = 2$ $y = 1/2$.

180. Prove or disprove: For all real numbers x and y, $\lfloor x + \lfloor x \rfloor \rfloor = \lfloor 2x \rfloor$.
Ans: False: $x = 1/2$.

181. Prove or disprove: For all real numbers x and y, $\lfloor xy \rfloor = \lfloor x \rfloor \cdot \lfloor y \rfloor$.
Ans: False: $x = 3/2$, $y = 3/2$.

182. Give a proof by cases that $x \leq |x|$ for all real numbers x.
Ans: Case 1, $x \geq 0$: then $x = |x|$, so $x \leq |x|$. Case 2, $x < 0$: here $x < 0$ and $0 < |x|$, so $x < |x|$.

183. Suppose you are allowed to give either a direct proof or a proof by contraposition of the following: if $3n + 5$ is even, then n is odd. Which type of proof would be easier to give? Explain why.
Ans: It is easier to give a contraposition proof; it is usually easier to proceed from a simle expression (such as n) to a more complex expression (such as $3n + 5$ is even). Begin by supposing that n is not odd. Therefore n is even and hence $n = 2k$ for some integer k. Therefore $3n + 5 = 3(2k) + 5 = 6k + 5 = 2(3k + 2) + 1$, which is not even. If we try a direct proof, we assume that $3n + 5$ is even; that is, $3n + 5 = 2k$ for some integer k. From this we obtain n = $(2k - 5)/3$, and it is not obvious from this form that n is even.

184. Prove that the following three statements about positive integers n are equivalent: (a) n is even; (b) $n^3 + 1$ is odd; (c) $n^2 - 1$ is odd.
Ans: Prove that (a) and (b) are equivalent and that (a) and (c) are equivalent.

185. Given any 40 people, prove that at least four of them were born in the same month of the year.
Ans: If at most three people were born in each of the 12 months of the year, there would be at most 36 people.

186. Prove that the equation $2x^2 + y^2 = 14$ has no positive integer solutions.
Ans: Give a proof by cases. There are only six cases that need to be considered: $x = y = 1$; $x = 1, y = 2$; $x = 1, y = 3$; $x = 2, y = 1$; $x = y = 2$; $x = 2, y = 3$.

187. What is wrong with the following "proof" that -3 = 3, using backward reasoning? Assume that -3 = 3. Squaring both sides yields $(-3)^2 = 3^2$, or 9 = 9. Therefore -3 = 3.
Ans: The steps in the "proof" cannot be reversed. Knowing that the squares of two numbers, -3 and 3, are equal does not allow us to infer that the two numbers are equal.