Problem: Transform the payment number and the date of the first payment into the corresponding month-number and year.

pn: Payment number $0 \le pn < n$, where n is the number of periods in the term.
month: A three-letter abbreviation for the month. Month abbreviations are jan, feb, mar, apr, may, jun, jul, aug, sep, oct, nov, dec.
m: Month number. Months are numbered from zero to eleven. $0 \le m \le 11$ The month-number for January is 0. The month-number for February is 1. The month-number for December is 11.
m_0: Initial month-number – the month when the first payment is due. Months are numbered from zero to eleven.
y: The year that a payment is due.
y_0: Initial year – the year when the first payment is due.

Example: Suppose the first payment is due April 5, 2005. Payments are due on the fifth of every month for a term of five years.

pn: $0 \le pn \le 59$
$m_0 = 3$
$y_0 = 2005$

Desired values for *y*, the year that a payment is due, and *m*, the number of the month that is payment is due, are given in Figure 1.

pn	*y*	month	*m*
	2005	jan	0
	2005	feb	1
	2005	mar	2
0	2005	apr	3
1	2005	may	4
2	2005	jun	5
3	2005	jul	6
4	2005	aug	7
5	2005	sep	8
6	2005	oct	9
7	2005	nov	10
8	2005	dec	11

pn	*y*	month	*m*
9	2006	jan	0
10	2006	feb	1
11	2006	mar	2
12	2006	apr	3
13	2006	may	4
14	2006	jun	5
15	2006	jul	6
16	2006	aug	7
17	2006	sep	8
18	2006	oct	9
19	2006	nov	10
20	2006	dec	11

pn	*y*	month	*m*
21	2007	jan	0
22	2007	feb	1
23	2007	mar	2
24	2007	apr	3
25	2007	may	4
26	2007	jun	5
27	2007	jul	6
28	2007	aug	7
29	2007	sep	8
30	2007	oct	9
31	2007	nov	10
32	2007	dec	11

Figure 1. Desired values for *m* and *y*.

Finding *m*, the number of the month that a payment is due: The first step is to add the payment number *pn* to the month-number that the first payment is due, m_0. The expression $(pn + m_0)$ produces a sequence $3, 4, \cdots, 59$. The problem here is that we have no months that have month-numbers greater than 11. We must keep the result of the expression for *m*, $0 \le m \le 11$. If we find the remainder of the expression $(pn + m_0)$ divided by 12, the number of months in a year, we guarantee that the result will be between 0 and 11. Now, will the expression, $m = (pn + m_0)\%12$ produce the desired result? Refer to Table 1 and observe that the result does, indeed, produce the desired result.

pn	*y*	month	*m*
33	2008	jan	0
34	2008	feb	1
35	2008	mar	2
36	2008	apr	3
37	2008	may	4
38	2008	jun	5
39	2008	jul	6
40	2008	aug	7
41	2008	sep	8
42	2008	oct	9
43	2008	nov	10
44	2008	dec	11

pn	*y*	month	*m*
45	2009	jan	0
46	2009	feb	1
47	2009	mar	2
48	2009	apr	3
49	2009	may	4
50	2009	jun	5
51	2009	jul	6
52	2009	aug	7
53	2009	sep	8
54	2009	oct	9
55	2009	nov	10
56	2009	dec	11

pn	*y*	month	*m*
57	2010	jan	0
58	2010	feb	1
59	2010	mar	2
	2010	apr	3
	2010	may	4
	2010	jun	5
	2010	jul	6
	2010	aug	7
	2010	sep	8
	2010	oct	9
	2010	nov	10
	2010	dec	11

Figure 1. Desired values for *m* and *y* (continued).

pn	*month*	$(pn+m_0)\%12$	*m*	$y_0+(pn+m_0)/12$	*y*
0	apr	(0+3)%12=3	3	2005+(0+3)/12=2005	2005
1	may	(1+3)%12=4	4	2005+(1+3)/12=2005	2005
…	…	…	…	…	…
8	dec	(8+3)%12=11	11	2005+(8+3)/12=2005	2005
9	jan	(9+3)%12=0	0	2005+(9+3)/12=2006	2006
10	feb	(10+3)%12=1	1	2005+(10+3)/12=2006	2006
…	…	…	…	…	…
20	dec	(20+3)%12=11	11	2005+(20+3)/12=2006	2006
21	jan	(21+3)%12=0	0	2005+(21+3)/12=2007	2007
22	feb	(22+3)%12=1	1	2005+(22+3)/12=2007	2007
…	…	…	…	…	…
32	dec	(32+3)%12=11	11	2005+(32+3)/12=2007	2007
33	jan	(33+3)%12=0	0	2005+(33+3)/12=2008	2008
34	feb	(34+3)%12=1	1	2005+(34+3)/12=2008	2008
…	…	…	…	…	…
44	dec	(44+3)%12=11	11	2005+(44+3)/12=2008	2008
45	jan	(45+3)%12=0	0	2005+(45+3)/12=2009	2009
46	feb	(46+3)%12=1	1	2005+(46+3)/12=2009	2009
…	…	…	…	…	…
56	dec	(56+3)%12=11	11	2005+(56+3)/12=2009	2009
57	jan	(57+3)%12=0	0	2005+(57+3)/12=2010	2010
58	feb	(57+3)%12=1	1	2005+(58+3)/12=2010	2010
59	mar	(57+3)%12=2	2	2005+(59+3)/12=2010	2010

Table 1. Proposed expressions for *m* and *y*.

Finding *y*, the year that a payment is due: The first step is to add the payment number *pn* to the month-number that the first payment is due, m_0. The expression $(pn + m_0)$ produces a sequence $3,4,\cdots,59$. Clearly, the sequence is nowhere close to what we desire. The sequence we desire is shown in Table 2.

$(pn + m_0)$	y	$(pn + m_0)$	y	$(pn + m_0)$	y
3	2005	12	2006	24	2007
4	2005	13	2006	25	2007
…	…	…	…	…	…
11	2005	23	2006	35	2007
$(pn + m_0)$	y	$(pn + m_0)$	y	$(pn + m_0)$	y
36	2008	48	2009	60	2010
37	2008	49	2009	61	2010
…	…	…	…	62	2010
47	2008	59	2009		

Table 2. $(pn + m_0)$ and y

The first insight is to recognize that the initial year, y_0, must be added to the expression yielding $y_0 + (pn + m_0)$. We must try to make the expression $(pn + m_0)$ serve as an addition to the initial year. We want an expression that will evaluate to 0 for values of the sum $3 \le (pn + m_0) \le 11$. In a similar way our desired expression must evaluate to 1 for $12 \le (pn + m_0) \le 23$ and 2 for $24 \le (pn + m_0) \le 35$ and so on. Using integer division by 12, we can achieve the desired values. Our desired expression becomes $y = y_0 + (pn + m_0)/12$