Number Systems

A number N is shown in Figure 1. Components of the number include the *integer-portion*, the *radix point*, the *fractional-portion*, and the *radix*.

$$(N)_r = (\textit{integer-portion}).(\textit{fractional-portion})_r$$

radix point **radix**

Figure 1. A Number *N*.

Example 1 shows a decimal number.

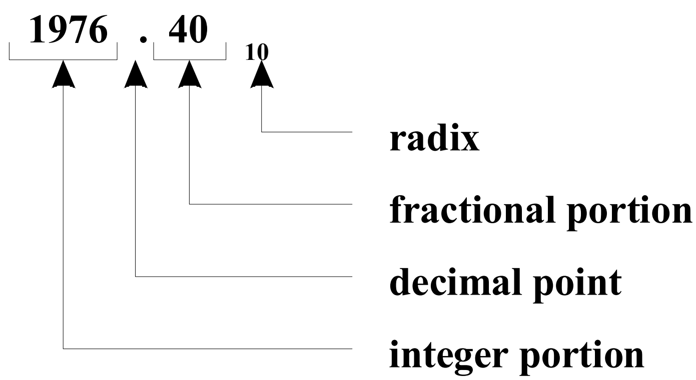

Figure 2. A decimal number.

Example 2 shows a binary number.

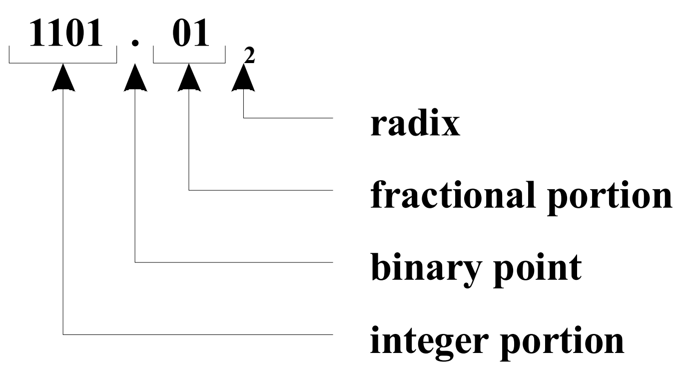

Figure 3. A binary number.

Juxtapostional Notation

juxtaposition means next to, or side by side

$$(N)_r = (a_{n-1}a_{n-2}\cdots a_1 a_0)\bullet(a_{-1}a_{-2}\cdots a_{-m})_r$$

r: is the radix of the number *N*.
a: is a digit in the set of digits defined for radix *r*.
n: is the number of digits in the integer portion.
m: is the number of digits in the fractional portion.
a_{n-1}: is the most significant digit
a_{-m}: is the least significant digit

Example: 1976.4_{10}

r: 10 (radix)
$a_j \in D, D = \{0,1,2,3,4,5,6,7,8,9\}$
$a_3 = 1, a_2 = 9.a_1 = 7, a_0 = 6, a_{-1} = 4$
n: number of digits in the integer portion (4).
m: number of digits in the fractional portion (1).
a_3: is the most significant digit (1).
a_{-1}: is the least significant digit (4).

Example: 1101.01_2
r: 2 (radix)
$a_j \in D, D = \{0,1\}$
$a_3 = 1, a_2 = 1, a_1 = 0, a_0 = 1, a_{-1} = 0, a_{-2} = 1$
n: number of digits in the integer portion (4).
m: number of digits in the fractional portion (2).
a_3: is the most significant digit (1).
a_{-2}: is the least significant digit (1).

Polynomial Notation

$$(N)_r = \sum_{j=-m}^{n-1} a_j r^j = a_{n-1}r^{n-1} + a_{n-2}r^{n-2} + \cdots + a_0 r^0 + a_{-1}r^{-1} + a_{-2}r^{-2} + \cdots a_{-m}r^{-m}$$

Example:

$(1101.101)_2$	=	**1**	×	$\mathbf{2^3}$	=	**8**
	+	**1**	×	$\mathbf{2^2}$	=	**4**
	+	**0**	×	$\mathbf{2^1}$	=	**0**
	+	**1**	×	$\mathbf{2^0}$	=	**1**
	+	**1**	×	$\mathbf{2^{-1}}$	=	**½**
	+	**0**	×	$\mathbf{2^{-2}}$	=	**0**
	+	**1**	×	$\mathbf{2^{-3}}$	=	**1/8**
						13 5/8

Example:

$(1976.4)_{10}$	=	**1**	×	$\mathbf{10^3}$	=	**1000**
	+	**9**	×	$\mathbf{10^2}$	=	**900**
	+	**7**	×	$\mathbf{10^1}$	=	**70**
	+	**6**	×	$\mathbf{10^0}$	=	**6**
	+	**4**	×	$\mathbf{10^{-1}}$	=	**4/10**
						1976 4/10

The Radix Divide/Multiply Method

Objective:	Covert a decimal number to an equivalent number in another radix (base).	
Solution:	**Step**	**Discussion**
	1	Separate the integer and fractional portions of the decimal number
	Integer portion conversion algorithm:	
	Step	**Discussion**
	0	Divide the decimal number by the radix. The remainder is a_i, i=0,1,2,3 … *n*-1. Quotient q_i becomes the dividend in the iteration. r : radix, divisor d_i : dividend in iteration *i*. d_0 is the initial dividend, the decimal number to be converted to the foreign base. q_i : quotient in iteration *i* a_i : remainder produced in iteration *i*, i^{th} digit of the number in radix *r*. $a_i = d_i \bmod r, d_{i+i} = \lfloor d_i \div r \rfloor$
	1 … *n*-1	Perform step 0 until the dividend equal zero.

Example: Convert 29_{10} to binary.

Step	Radix-Divisor	Dividend	Quotient	Remainder	a_i
0	2	29	14	1	a_0
1	2	14	7	0	a_1
2	2	7	3	1	a_2
3	2	3	1	1	a_3
4	2	1	0	1	a_4
5	2	0 **stop!**			

$29_{10} = 11101_2$

$\mathbf{11101_2}$	=		**1**	×	$\mathbf{2^4}$	=	**16**
		+	**1**	×	$\mathbf{2^3}$	=	**8**
		+	**1**	×	$\mathbf{2^2}$	=	**4**
		+	**0**	×	$\mathbf{2^1}$	=	**0**
		+	**1**	×	$\mathbf{2^0}$	=	**1**
							29

	Fractional portion conversion algorithm:	
	Step	**Discussion**
	-1	Multiply the decimal fraction by the radix. The integer portion of the product is a_{-i}, i=-1,-2, … -m
	-2 … -*m*	Remove the integer portion of the product obtained in the previous step. The remaining decimal fraction is the multiplicand for this step. $\text{fraction}_{i+1}=\text{frac}(\text{product}_i)$ Repeat step –1 and the foregoing until either $\text{fraction}_{i+1}=0$ or the desired precision is obtained.

Example: Convert 20.1_{10} to binary.

Step	**Radix-Multiplier**	**Fraction-Multiplicand**	**Product**	**Integer Portion**	a_i
-1	2	0.1	0.2	0	a_{-1}
-2	2	0.2	0.4	0	a_{-2}
-3	2	0.4	0.8	0	a_{-3}
-4	2	0.8	1.6	1	a_{-4}
-5	2	0.6	1.2	1	a_{-5}
-6	2	0.2	0.4	0	a_{-6}
-7	2	0.4	0.8	0	a_{-7}
-8	2	0.8	1.6	1	a_{-8}

$$0.1_{10} \approx 0.00011001_2$$

$\mathbf{0.00011001_2}$	=		**0**	×	$\mathbf{2^{-1}}$	=	
		+	**0**	×	$\mathbf{2^{-2}}$	=	
		+	**0**	×	$\mathbf{2^{-3}}$	=	
		+	**1**	×	$\mathbf{2^{-4}}$	=	**1/16**
		+	**1**	×	$\mathbf{2^{-5}}$	=	**1/32**
		+	**0**	×	$\mathbf{2^{-6}}$	=	
		+	**0**	×	$\mathbf{2^{-7}}$	=	
		+	**1**	×	$\mathbf{2^{-8}}$	=	**1/256**
							0.09765625

Binary, Octal, and Hexadecimal

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$2^3 = 8$ Octal

$$(N)_2 \rightarrow (N)_8$$

$110101001.111_2=651.7_8$

110	101	001	.	111
6	5	1	.	7

$2^4 = 16$ Hexadecimal

$$(N)_2 \rightarrow (N)_{16}$$

$110101001.111_2=651.7_8$

0001	1010	1001	.	111***0***
1	A	9	.	E

Tricks of the Trade

Speed up binary-to-decimal and decimal-to-binary conversions by converting to and from hexadecimal first.

$$(N)_2 \rightarrow (N)_{16} \rightarrow (N)_{10}$$

$$(N)_{10} \rightarrow (N)_{16} \rightarrow (N)_2$$

By dividing the problem into two parts fewer substitution, multiplications, and divisions are performed. In fact, four (4) times fewer operations are performed.