Definition 1

Let S be the sum of the geometric sequence

$$S = a + ar + ar^2 + \cdots + ar^n$$

$$S = \sum_{k=0}^{n} ar^k$$

$$S = \frac{a(r^{n+1} - 1)}{r - 1}, r \neq 1$$

Derivation:

$$S = a + ar + ar^2 + \cdots + ar^n$$

$$rS = ar + ar^2 + ar^3 \cdots + ar^n + ar^{n+1}$$

$$rS - S = ar^{n+1} - a$$

$$S(r - 1) = a(r^{n+1} - 1)$$

$$S = \frac{a(r^{n+1} - 1)}{r - 1}$$

EXAMPLE 1

Find the sum of the sequence $1 + 2 + 4 + 8 + 16 + 32$.

Solution:

$$S = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$

$$S = \frac{a(r^{n+1} - 1)}{r - 1}$$

$$S = \frac{1(2^{5+1} - 1)}{2 - 1} = 2^6 - 1 = \mathbf{63}$$

Definition 2

Let S be the sum of the geometric sequence

$$S = a + ar + ar^2 + \cdots + ar^{n-1}$$

$$S = \sum_{k=0}^{n-1} ar^k$$

$$S = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

EXAMPLE 2.1

Find the sum of the sequence

$$S = R + R(1 + i) + R(1 + i)^2 + R(1 + i)^2 + \cdots + R(1 + i)^{n-1}$$

Solution:

$$S = R\frac{(1 + i)^n - 1}{(1 + i) - 1} = R\frac{(1 + i)^n - 1}{i}$$

$$\boldsymbol{S = R\frac{(1 + i)^n - 1}{i}}$$

EXAMPLE 2.2 Find the sum of the compounded amounts resulting from periodic deposits into an interest bearing account. Each deposit, $R = \$300$. Deposits are made every month for 30 years. The interest rate on the account is 6%.

Solution:

1. $f = 12$: monthly
2. $r = \frac{6}{100} = 0.06$: annual interest rate
3. $i = \frac{r}{f} = \frac{0.06}{12} = 0.005$: monthly interest rate
4. $T = 30$: number of years in the term
5. $n = f \cdot T = 12 \cdot 30 = 360$
6. $R = \$300$: the monthly deposit
7. $\boldsymbol{S = R\frac{(1+i)^n - 1}{i} = \$300\frac{(1+0.005)^{360} - 1}{0.005} = \$301,354.51}$

The amount of money actually deposited was equal to the number of payments times the amount deposited, $360 \times 300 = \$108{,}000$.
The compound interest $I = S - P = \$301{,}354.51 - 108{,}000.00 = \$193{,}354.51$