1. Print your name on your scantron in the space labeled NAME.
2. Print CMSC 2123 in the space labeled SUBJECT.
3. Print the date 12-12-2013, in the space labeled DATE.
4. Print your CRN, 11786, in the space labeled PERIOD.
5. Print the test number and version, T3/V1, in the space labeled TEST NO.
6. Mark the best selection that satisfies the question. If selection b is better than selections a and d, then mark selection b. Mark only one selection.
7. Darken your selections completely. Make a heavy black mark that completely fills your selection.
8. Answer all 50 questions.
9. Record your answers on SCANTRON form 882-E (It is green!)
10. Submit your completed scantron on Thursday, December 12, 2013 at 11:00 a.m. in MCS 115.
1. (1.1 Propositional Logic) Let \( p \) and \( q \) be the propositions “Swimming at the New Jersey shore is allowed” and “Toxic waste has polluted the shoreline,” respectively. Which compound proposition is NOT expressed properly as an English sentence?

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sim p \lor q )</td>
<td>Swimming at the New Jersey shore is not allowed or toxic waste has polluted the shoreline.</td>
</tr>
<tr>
<td>( p \rightarrow \sim q )</td>
<td>if swimming at the New Jersey shore is allowed then toxic waste has not polluted the shoreline.</td>
</tr>
<tr>
<td>( p \leftrightarrow \sim q )</td>
<td>Swimming at the New Jersey shoreline is allowed if and only if toxic waste has polluted the shoreline.</td>
</tr>
<tr>
<td>( \sim p \lor (p \land q) )</td>
<td>Either swimming at the New Jersey shore is not allowed, or else swimming at the New Jersey shore is allowed and toxic waste has polluted the shoreline.</td>
</tr>
</tbody>
</table>

2. (1.1 Propositional Logic) Determine which compound proposition is true.

   a. \( 2 + 2 = 4 \) if and only if \( 2 + 3 = 4 \).
   b. if \( 1 + 1 = 3 \), then \( 2 + 3 = 4 \).
   c. \( 2 + 2 = 5 \) or \( 1 + 3 = 2 \).
   d. \( 2 + 2 = 4 \) and if \( 1 + 3 = 2 \).

3. (1.2 Propositional Equivalences) Use a truth table to determine if the following is a tautology, a contradiction, or neither.

   \[
   [(p \rightarrow q) \land p] \rightarrow q
   \]

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

   a. tautology
   b. contradiction
   c. neither

4. (1.2 Propositional Equivalence) Which of the following are NOT tautologies?

   a. \( p \leftrightarrow q \equiv ((p \land q) \lor (\sim p \land \sim q)) \)
   b. \( \sim (p \land q) \equiv \sim p \lor \sim q \)
   c. \( p \leftrightarrow q \equiv ((p \rightarrow q) \land (q \rightarrow p)) \)
   d. \( (p \rightarrow q) \rightarrow r \equiv p \rightarrow (q \rightarrow r) \)
5. (1.3 Predicates and Quantifiers) Let $P(x)$ be the statement “$x$ can speak Russian” and let $Q(x)$ be the statement “$x$ knows the computer language C++.” Which of the expressions given below accurately represents the statement “There is a student at your school who can speak Russian but who doesn’t know C++.”?

a. $\exists x(P(x) \land Q(x))$

b. $\exists x(P(x) \land \neg Q(x))$

c. $\forall x(P(x) \lor Q(x))$

d. $\forall x(\neg(P(x) \lor Q(x)))$

6. (1.3 Predicates and Quantifiers) Let $C(x)$ be the statement “$x$ is a comedian” and let $F(x)$ be the statement “$x$ is funny.” Select the correct translation of the expression $\exists x(C(x) \land F(x))$.

a. Every comedian is funny.

b. Every person is a funny comedian.

c. There exists a person such that if she or he is a comedian, then she or he is funny.

d. Some comedians are funny.

7. (1.4 Nested Quantifiers) Let $I(x)$ be the statement “$x$ has an Internet connection” and $C(x, y)$ be the statement “$x$ and $y$ have chatted over the Internet,” where the domain for the variables $x$ and $y$ consists of all students in your class. Select the expression that is the translation of the statement “Everyone in your class with an Internet connection has chatted over the Internet with at least one other student in your class.”

a. $\forall x(I(x) \rightarrow \exists y(x \neq y \land C(x, y)))$

b. $\exists x(I(x) \land \forall y(x \neq y \rightarrow C(x, y)))$

c. $\forall x(I(x) \leftrightarrow \exists y(x \neq y \land C(x, y)))$

d. $\exists x(I(x) \lor \forall y(x \neq y \rightarrow C(x, y)))$

8. (1.4 Nested Quantifiers) Select the alternative where the statement shown was rewritten so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

$\neg(\exists x \exists y \neg P(x, y) \land \forall x \forall y Q(x, y))$

a. $(\forall x \forall y P(x, y)) \land (\exists x \exists y \neg Q(x, y)))$

b. $(\exists x \exists y P(x, y)) \lor (\forall x \forall y \neg Q(x, y)))$

c. $(\forall x \forall y P(x, y)) \lor (\exists x \exists y \neg Q(x, y)))$

d. $(\exists x \exists y P(x, y)) \land (\forall x \forall y \neg Q(x, y)))$
9. (1.5 Rules of Inference) The following argument form is called:

\[ p \rightarrow q \]
\[ q \rightarrow r \]
\[ \therefore p \rightarrow r \]

a. modus tollens
b. modus ponens
c. disjunctive syllogism
d. hypothetical syllogism

10. (1.5 Rules of Inference) What rule of inference is used in the argument given below?

If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

a. Addition
b. Simplification
c. Modus ponens
d. Modus tollens.

11. (2.2 Set operations) Two sets are disjoint if:

a. \( A \cap B = \emptyset \)
b. \( |A| = |B| \)
c. \( A \cap B = B \cap A \)
d. \( A \cup (A \cap B) = A \)

12. (2.2 Set operations) What set does the Venn diagram given below represent?

a. \( (X - Y) \cup (X - Z) \)
b. \( (X \cap Y) \cup (X \cap Z) \)
c. \( (X \cup Y) \cap (X \cup Z) \)
d. \( (X \cap Y) + (X \cap Z) \)
13. (2.2 Set operations) Let $X$, $Y$ and $Z$ be sets. Which of the following is an example of the law of commutativity?

a. $(X \cup Y) \cup Z = X \cup (Y \cup Z)$
b. $(X \cap Y) \cup Z = (Y \cap X) \cup Z$
c. $(X \cap Y) \cup Z = Z \cup (X \cap Y)$
d. Both choices $b$ and $c$.

14. (2.2 Set operations) Which of the following sets is described by the Venn diagram shown below:

![Figure 14. Venn Diagram for Question 14.](image)

a. $\overline{C} \cup (A \cup B)$
b. $C \cap (A \cup B)$
c. $(C \cap \overline{A}) \cup (C \cap \overline{B})$
d. $C \cup (A \cup B)$

15. (2.3 Functions) Which of the following functions does NOT have an inverse?

a. $X = \{x \in \mathbb{Z}|-2 < x \leq 5\}, \ Y = \{y \in \mathbb{Z}|1 < y \leq 8\}, f: X \to Y$ defined by $f(x) = x + 3$ for all $x \in X$.
b. $f: \mathbb{Z} \to \mathbb{Z}, f(x) = 4x + 7$ for all $x \in \mathbb{Z}$
c. $f: \mathbb{R} \to \mathbb{R}, f(x) = 2x - 5$ for all $x \in \mathbb{R}$.
d. $f: \mathbb{Q} \to \mathbb{Q}, f(x) = 8x$ for all $x \in \mathbb{Q}$.

16. (2.3 Functions) Which of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ is NOT one-to-one?

a. $f(n) = n - 1$
b. $f(n) = n^3$
c. $f(n) = n^2 + 1$
d. $f(n) = -3n + 4$
17. (2.3 Functions) Let $A = \{1,2,3,4\}$ and $B = \{a,b,c,d\}$ be sets. Which of the following arrow diagrams is NOT a function from $A$ into $B$?

![Arrow Diagrams](image)

18. (2.4 Sequences and Summations) Select the formula that is used to produce the integer sequence

$2, 16, 54, 128, 250, 432, 686, \cdots$

a. $a_n = 2 + 14n, n = 0,1,2,3,\cdots$

b. $a_n = 2n^3, n = 0,1,2,3,\cdots$

c. $a_n = 2n^3, n = 1,2,3,\cdots$

d. $a_n = 4 + 14(n - 1) - 2, n = 1,2,3,\cdots$

19. (2.4 Sequences and Summations) Evaluate

$$\sum_{i=0}^{9} \sum_{j=0}^{4} (3i + 2j)$$

a. 875

b. 1025

c. 975

d. 1125
20. (2.4 Sequences and Summations) The first ten integers of a sequence are given below. Find the sixteenth integer in the sequence.

6, 16, 23, 42, 52, 62, 72, 106, 119, 132, ...

a. 184  
b. 197  
c. 258  
d. 274

21. (3.1 Algorithms) What is the time complexity of the binary search algorithm?

a. $O(\log n)$  
b. $O(1)$  
c. $O(n)$  
d. $O(n^2)$

22. (3.2 The Growth of Functions) Select a valid relationship between functions $g(x)$ and $h(x)$ in the diagram below?

\begin{center}
\textbf{Figure 22 Diagram for Question 22}
\end{center}

a. $g(x)$ is $\Omega(h(x))$  
b. $g(x)$ is $O(h(x))$  
c. $g(x)$ is $\Theta(h(x))$  
d. $g(x)$ is $\Phi(h(x))$
23. (3.2 The Growth of Functions) Select a valid relationship between functions $f(x)$ and $g(x)$ in the diagram below?

diagram

- a. $g(x)$ is $O(f(x))$
- b. $g(x)$ is $\Theta(f(x))$
- c. $g(x)$ is $\Omega(f(x))$
- d. $g(x)$ is $\Phi(f(x))$

Figure 23  Diagram for Question 23
24. (3.3 Complexity of Algorithms) Find the time complexity function $f(n)$ for the code fragment in the Figure 24.1 below. Feel free to employ the table in Figure 24.2 to assist you in your computation of function $f(n)$.

```java
int sum=0;
for(int i=0;i<n;i++) {
    for(int j=0;j<i*i;j++) {
        sum++;
    }
}
```

**Figure 24.1 Code fragment for Question 24**

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>int sum=0;</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>int i=0;</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>while (i&lt;n) {</td>
<td></td>
</tr>
<tr>
<td></td>
<td>int j=0;</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>while (j&lt;i*i) {</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>sum++;</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>j++;</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>}</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>i++;</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>}</td>
<td></td>
</tr>
</tbody>
</table>

`Total` $f(n) =$

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Cost</th>
</tr>
</thead>
</table>

`Total` $f(n) =$

`Total` $f(n) =$

**Figure 24.2 Expanded Code fragment for Question 24**
a. \( f(n) = 3n^2 + 4n + 3 \)
b. \( f(n) = \frac{3}{2}n^2 + \frac{11}{2}n + 3 \)
c. \( f(n) = f(n) = \frac{4}{3}n^3 - 2n^2 + \frac{14}{3}n + 3 \)
d. \( f(n) = n^3 + \frac{3}{2}n^2 + \frac{25}{6}n + 3 \)

Questions 26, 27, and 28 relate to the timing function \( f(x) = 3n\lfloor \log_2 n \rfloor + 4n + 3 \) of an algorithm.

25. (3.1 The Growth of Functions) Find the best function \( g(x) \) such that \( f(x) \) is \( O(g(x)) \).

a. \( g(x) = n \)
b. \( g(x) = n^2 \)
c. \( g(x) = 2^n \)
d. \( g(x) = n\lfloor \log_2 n \rfloor \)

26. (3.1 The Growth of Functions) Find the best value for constant \( C \) in the definition of Big-O notation \( f(x) \) is \( O(g(x)) \) where \( g(x) \) was determined in the previous question.

a. \( C = 3 \)
b. \( C = \lim_{n \to \infty} \frac{f(x)}{g(x)} \)
c. \( C = 4 \)
d. \( C = \lim_{n \to \infty} \frac{3\lfloor n \log_2 n \rfloor + 4n + 3}{n^2} \)

27. (3.1 The Growth of Functions) Find the smallest integer value for constant \( k \) in the definition of Big-O notation \( f(x) \) is \( O(g(x)) \) where \( g(x) \) and constant \( C \) were determined in the previous questions.

a. \( k = 3 \)
b. \( k = 18 \)
c. \( k = 32 \)
d. \( k = 4 \)

28. (3.4 The Integers and Division) What are the quotient and remainder when -111 is divided by 11?

a. \( q = 10, r = 1 \)
b. \( q = -11, r = 10 \)
c. \( q = -10, r = -1 \)
d. \( q = -10, r = -0.09 \)

29. (3.4 The Integers and Division) Select the statement that is true.

a. \(-122 \equiv 5 \text{ mod } 17\)
b. \(80 \equiv 5 \text{ mod } 17\)
c. \(-29 \equiv 5 \text{ mod } 17\)
d. \(103 \equiv 5 \text{ mod } 17\)
30. (3.5 Primes and the Greatest Common Divisor) Find the gcd(48,116).
   a. 16  
   b. 4  
   c. 8  
   d. 2

31. (3.5 Primes and the Greatest Common Divisor) What is the least common multiple of $2^2 \cdot 3^3 \cdot 5^5$ and $2^5 \cdot 3^3 \cdot 5^2$
   a. $2^2 \cdot 3^3 \cdot 5^2$  
   b. $2 \cdot 3 \cdot 5$  
   c. $2^5 \cdot 3^3 \cdot 5^5$  
   d. $2^5 \cdot 3^5 \cdot 5^5$

32. (3.6 Integers and Algorithms) Convert the decimal number 759 to a 12-bit binary number.
   a. 0010 1111 0111  
   b. 0010 1111 0101  
   c. 0001 1011 0111  
   d. 0001 1101 1111

33. (3.6 Integers and Algorithms) Compute the binary difference 1100101 - 11011. Both binary numbers are unsigned.
   a. 1001010  
   b. 101010  
   c. 1001110  
   d. 110101

34. (3.6 Integers and Algorithms) Find the two’s complement of the 7-bit binary number 101110.
   a. 010001  
   b. 01110  
   c. 101110  
   d. 010010

35. (4.1 Mathematical Induction) Use induction to prove that $P(n)$: $\sum_{i=1}^{n} (2n - 1) = n^2 \ \forall \ n \geq 1$. What is the first step in proving $P(n)$ true.
   a. Assume that $P(2), P(3), \ldots, P(k)$ is true.  
   b. Prove $\sum_{i=1}^{k} (2 \times 1 - 1) = 1^2$  
   c. Prove $\sum_{i=1}^{k} (2(k + 1) - 1) = (k + 1)^2$  
   d. Prove $P(k + 1)$ is true
36. (4.1 Mathematical Induction) Use induction to prove that
\[ P(n): \sum_{i=1}^{n} (2n - 1) = n^2 \quad \forall n \geq 1 \]
What is the name of the first step in proving \( P(n) \) true.

a. Inductive hypothesis  
   b. Inductive step  
   c. Deductive hypothesis  
   d. Basis step.

37. (4.1 Mathematical Induction) Use induction to prove that \( P(n): \sum_{i=1}^{n} (2n - 1) = n^2 \quad \forall n \geq 1 \)
What is the inductive step in proving \( P(n) \) true.

a. Prove \( \sum_{i=1}^{1} (2 \times 1 - 1) = 1^2 \)  
   b. Assume that \( P(2), P(3), \ldots, P(k) \) is true and prove \( P(k+1) \) is true.
   c. Prove \( P(k) \) is true  
   d. Assume \( \sum_{i=1}^{k} (2k - 1) = k^2 \) and prove \( \sum_{i=1}^{k+1} (2(k+1) - 1) = (k+1)^2 \)

38. (Mathematical Induction) Use induction to prove that \( P(n): \sum_{i=1}^{n} (2n - 1) = n^2 \quad \forall n \geq 1 \)
What is the Inductive Hypothesis in proving \( P(n) \)?

a. Prove \( \sum_{i=1}^{1} (2 \times 1 - 1) = 1^2 \)  
   b. Prove \( \sum_{i=1}^{k+1} (2(k+1) - 1) = (k+1)^2 \)  
   c. Assume that \( P(k) \) is true.  
   d. Prove \( P(k+1) \) is true

39. (4.3 Recursive Definition and Structural Induction) Find a formula for \( f(n) \) when \( n \) is a nonnegative integer given the recursive definition shown below.
\[ f(0) = 2, f(1) = 3, f(n) = f(n-1) - 1 \quad \text{for } n \geq 2 \]

a. \( f(n) = 4 - n, n \geq 1 \)  
   b. \( f(n) = 2 - n, n \geq 1 \)  
   c. \( f(n) = 3 - n, n \geq 1 \)  
   d. \( f(n) = 1 - n, n \geq 1 \)

40. (4.4 Recursive Algorithms) Find a recursive algorithm for finding the sum of the first \( n \) positive integers.

```
int sum(int n){return n==1?1:(n-1)+sum(n);}
```

Figure 40 a

```
int sum(int n){return n==1?1:sum(n-1)+n;}
```

Figure 40 b

```
int sum(int n){return n==1?1:sum(n-1)*n;}
```

Figure 40 c

```
int sum(int n){return n==1?1:(n-1)*sum(n);}
```

Figure 40 d

41. (5.1 The Basics of Counting) How many bit strings of length eight start with 11 or end with 00?

a. 64  
   b. 112
42. (5.1 The Basics of Counting) How many bit strings of length six do not have two consecutive 1s?
   a. 64
   b. 32
   c. 15
   d. 21

43. (5.2 The Pigeonhole Principle) How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 15} to guarantee that at least one pair of these numbers add up to 16?
   a. 5
   b. 6
   c. 3
   d. 4

44. (5.2 The Pigeonhole Principle) What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?
   a. 5001
   b. 101
   c. 4951
   d. 51

45. (5.3 Permutations and Combinations) There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?
   a. 720
   b. 6
   c. 36
   d. 46,655

46. (5.3 Permutations and Combinations) In how many ways can a set of five letters be selected from the English alphabet?
   a. \( P(26,5) \)
   b. 65,780
   c. \( \binom{5}{26} \)
   d. \( \frac{5!}{5!26!} \)
47. (5.4 Binomial Coefficients) What is the coefficient of $x^5y^8$ in $(x + y)^{13}$?

a. $\binom{13}{5} = \frac{13!}{(13-5)!}$
b. $\binom{13}{8} = \frac{13!}{(13-8)!}$
c. $\binom{13}{5} = \frac{13!}{5!(13-5)!}$
d. $\binom{13}{8} = \frac{13!}{8!(13-8)!}$

48. (5.4 Binomial Coefficients) What is the coefficient of $x^5y^{12}$ in $(3x - 2y)^{17}$?

a. $\frac{17!}{12!5!}3^{12}2^5$
b. $\frac{17!}{12!5!}3^52^{12}$
c. $-\frac{17!}{12!5!}3^{12}2^5$
d. $-\frac{17!}{12!5!}3^52^{12}$

49. (5.5 Generalized Permutations and Combinations) Every day a student randomly chooses a sandwich for lunch from a pile of wrapped sandwiches. If there are six kinds of sandwiches, how many different ways are there for the student to choose sandwiches for the seven days of a week if the order in which the sandwiches are chosen matters?

a. $\binom{7}{6}$
b. $7^6$
c. $\frac{7!}{(7-6)!}$
d. $6^7$

50. (5.5 Generalized Permutations and Combinations) How many ways are there to select five unordered elements from a set with three elements when repetition is allowed?

a. $\binom{7}{3}$
b. $3^7$
c. $C(7,2)$
d. $P(7,2)$