Definition 1

**ANNUITY.** An annuity is a sequence of equal payments made at equal intervals of time.

Definition 2

**AMOUNT of an ANNUITY.** Consider an ordinary annuity of $1000 per year for 4 years with money worth 5%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Payment</th>
<th>Interest</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1000</td>
<td>0</td>
<td>$1000</td>
</tr>
<tr>
<td>1</td>
<td>$1000</td>
<td>0.05($1000)</td>
<td>$1050</td>
</tr>
<tr>
<td>2</td>
<td>$1000</td>
<td>0.05($1050)</td>
<td>$1102.50</td>
</tr>
<tr>
<td>3</td>
<td>$1000</td>
<td>0.05($1050)</td>
<td>$1102.50</td>
</tr>
<tr>
<td>4</td>
<td>$1000</td>
<td>0.05($1050)</td>
<td>$1102.50</td>
</tr>
</tbody>
</table>

The amount $S$ of the annuity is the sum of the compound amounts of the several payments each accumulated to the end of the term. Since the first payment earns interest for 3 years, the second payment for 2 years, the third for 1 year, and the fourth is cash,

$$S = 1000(1.05)^3 + 1000(1.05)^2 + 1000(1.05) + 1000$$

or, reversing the order.

$$S = 1000 + 1000(1.05) + 1000(1.05)^2 + 1000(1.05)^3$$

$$S = 1000 \cdot \frac{1.05^4 - 1}{0.05}$$

In general

- $R$: is the periodic payment of an annuity.
- $i$: is the periodic interest rate.
- $n$: is the number of periods in the term.
- $S$: is the amount of the annuity (the future value)
- $A$: is the present value of the annuity

**Amount**

$$S = R \cdot \frac{(1 + i)^n - 1}{i}$$

**Present Value**

$$A = R \cdot \frac{1 - (1 + i)^{-n}}{i}$$

**Payment**

$$R = A \cdot \frac{i}{1 - (1 + i)^{-n}}$$
EXAMPLE 2.1 \hspace{1cm} Find the sum of the compounded amounts resulting from periodic deposits into an interest bearing account. Each deposit, $R = \$300$. Deposits are made every month for 30 years. The interest rate on the account is 6%.

Solution:

1. $f = 12$: monthly
2. $r = \frac{6}{100} = 0.06$: annual interest rate
3. $i = \frac{r}{f} = \frac{0.06}{12} = 0.005$: monthly interest rate
4. $T = 30$: number of years in the term
5. $n = f \cdot T = 12 \cdot 30 = 360$
6. $R = \$300$: the monthly deposit
7. $S = R \frac{(1+i)^n - 1}{i} = \$300 \frac{(1+0.005)^{360} - 1}{0.005} = \$301,354.51$