Definition 1

Let $S$ be the sum of the geometric sequence

$$S = a + ar + ar^2 + \cdots + ar^n$$

$$S = \sum_{k=0}^{n} ar^k$$

$$S = \frac{a(r^{n+1} - 1)}{r - 1}, r \neq 1$$

Derivation:

$$S = a + ar + ar^2 + \cdots + ar^n$$

$$rS = ar + ar^2 + ar^3 + \cdots + ar^n + ar^{n+1}$$

$$rS - S = ar^{n+1} - a$$

$$S(r - 1) = a(r^{n+1} - 1)$$

$$S = \frac{a(r^{n+1} - 1)}{r - 1}$$

Example 1

Find the sum of the sequence $1 + 2 + 4 + 8 + 16 + 32$.

Solution:

$$S = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$

$$S = \frac{a(r^{n+1} - 1)}{r - 1}$$

$$S = \frac{1(2^5+1 - 1)}{2 - 1} = 2^6 - 1 = 63$$

Definition 2

Let $S$ be the sum of the geometric sequence

$$S = a + ar + ar^2 + \cdots + ar^{n-1}$$

$$S = \sum_{k=0}^{n-1} ar^k$$

$$S = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

Example 2.1

Find the sum of the sequence

$$S = R + R(1 + i) + R(1 + i)^2 + R(1 + i)^2 + \cdots + R(1 + i)^{n-1}$$

Solution:

$$S = R \frac{(1 + i)^n - 1}{(1 + i) - 1} = R \frac{(1 + i)^n - 1}{i}$$

$$S = R \frac{(1 + i)^n - 1}{i}$$
**EXAMPLE 2.2** Find the sum of the compounded amounts resulting from periodic deposits into an interest bearing account. Each deposit, $R = $300. Deposits are made every month for 30 years. The interest rate on the account is 6%.

**Solution:**

1. $f = 12$: monthly
2. $r = \frac{6}{100} = 0.06$: annual interest rate
3. $i = \frac{r}{f} = \frac{0.06}{12} = 0.005$: monthly interest rate
4. $T = 30$: number of years in the term
5. $n = f \cdot T = 12 \cdot 30 = 360$
6. $R = $300: the monthly deposit
7. $S = R \frac{(1+i)^n-1}{i} = \frac{300(1+0.005)^{360}-1}{0.005} = $301,354.51

The amount of money actually deposited was equal to the number of payments times the amount deposited, $360 \times 300 = $108,000.

The compound interest $l = S - P = 301,354.51 - 108,000.00 = $193,354.51