1. Print your name on your scantron in the space labeled NAME.
2. Print CMSC 2123 in the space labeled SUBJECT.
3. Print the date, 5-2-2011, in the space labeled DATE.
4. Print your CRN, 21858, in the space labeled PERIOD.
5. Print the test number and version, T1/V1, in the space labeled TEST NO.
6. You may not consult your neighbors, colleagues, or fellow students to answer the questions on this test.
7. Mark the best selection that satisfies the question.
   7.1. **True/False:** Mark selection a if the statement is true or selection b if the statement is false.
   7.2. **Multiple Choice:** If selection b is better that selections a and d, then mark selection b. Mark only one selection. Please note that there are five possible answers for question 49 unlike other questions on the test that have only four possible answers.
8. Darken your selections completely. Make a heavy black mark that completely fills your selection.
9. Answer all 50 questions.
10. Record your answers on SCANTRON form 882-E (It is green!)
11. Submit your completed scantron to your instructor on Monday, May 2, 2011 at 11:00 a.m. in room MCS 113.
1. Let $X = \{1,3,5,7,9\}$ and $Y = \{2n - 1|n \in \mathbb{N}\}$, then $X \subset Y$.
   a. True
   b. False

2. The Well-Ordering Principle can be used to prove that the set of integers has a minimum element.
   a. True
   b. False

3. If $a|b$, then $\gcd(a, b) = b$.
   a. True
   b. False

4. The decimal number 12 is represented by the hexadecimal symbol C.
   a. True
   b. False

5. The decimal number 42 is equivalent to the binary number 101010.
   a. True
   b. False

6. The sum of the unsigned binary numbers $1010111 + 111001 = 10010000$.
   a. True
   b. False

7. A loop invariant is a set of statements that are false when a loop is entered and become true on the final exit from the loop.
   a. True
   b. False

8. If $p$ is prime and $p|ab$ where $a$ and $b$ are positive integers, then $p|a$ and $p|b$.
   a. True
   b. False

9. Assume $A$ is any set and $P(A)$ is the power set under the inclusion relation. Then $A$ is an upper bound and $\emptyset$ is a lower bound for any subset of $P(A)$.
   a. True
   b. False

10. Let $f = \{(x, 2x^3 + 3)|x \in \mathbb{Z}\}$, where $\mathbb{Z}$ is the set of integers. Then the range of $f$, $\text{Im}(f)$, is $\mathbb{Z}$.
    a. True
    b. False

11. Let $X$ be a finite set, and let $f: x \rightarrow x$ be a function. If $f$ is one-to-one and $\text{Im}(f) = X$, then $f$ is a one-to-one correspondence.
    a. True
    b. False
12. If \( x = -3.98 \), then the floor of \( x \), \( \lfloor x \rfloor \), is \( -3 \).
   a. True
   b. False

13. The linear congruence \( 9x \equiv 1 \pmod{36} \) has a solution.
   a. True
   b. False

14. The solution of the recurrence relation \( a_n = 2a_{n-1} - a_{n-2} \), where \( n \geq 2 \), with initial conditions \( a_0 = -3 \) and \( a_1 = 1 \) is \( a_n = -4 + 3n \).
   a. True
   b. False

15. Let \( f \) be a function of \( n \), where \( n \) is the size of the list to be processed. The term \textit{asymptotic} refers to the study of functions where are unaffected by the size of \( n \).
   a. True
   b. False

16. Let \( f(x) \) and \( g(x) \) be real-valued functions. Then \( f(x) = O(g(x)) \) if there exists positive constants, \( c \), and \( x_0 \), such that \( |f(x)| \leq c|g(x)| \) \( \forall x \geq x_0 \).
   a. True
   b. False

17. Let \( f(n) = \Theta(g(n)) \), then \( f(n) = \Theta(h(n)) \)
   a. True
   b. False

18. The selection sort efficiency is \( O(n^2) \)
   a. True
   b. False

19. Let \( X = \{-3, -2, -1, 0, 1, 2\}, Y = \{3, 4, 5, 6, 7\} \), and \( f = \{(-2, 3), (-1, 6), (0, 4), (1, 5), (2, 7)\} \). \( f \) is a function.
   a. True
   b. False

20. Consider the function \( f: S \rightarrow \mathbb{Z} \), where \( f = \{(x, x^2) | x \in S\}, S = \{-3, -2, -1, 0, 1, 2, 3\} \). \( f \) is onto \( \mathbb{Z} \).
   a. True
   b. False
21. Let $A$ and $B$ be subsets of the universal set $U$. Of which law is $A' \cup B' = (A \cap B)'$ an example?
   a. Absorptive law
   b. Distributive law
   c. DeMorgan’s law
   d. Idempotency law

22. Which of the following is true?
   a. 2 is a prime number and -5 is a nonnegative number.
   b. 5 is a negative integer if and only if pigs can fly.
   c. 12 is a prime number or -4 is not a negative number
   d. If 3 is not an even integer, then -3 is not a negative integer.

23. To determine whether the argument given below is valid, you would need to construct a truth table for which of the following?
   \[
   p \rightarrow (q \lor r) \\
   p \rightarrow \sim q \\
   \therefore p \lor r
   \]
   a. $(p \rightarrow (q \lor r)) \land (p \rightarrow \sim q) \lor (p \lor r)$
   b. $(p \rightarrow (q \lor r)) \land (p \rightarrow \sim q) \rightarrow (p \lor r)$
   c. $(p \rightarrow (q \lor r)) \rightarrow (p \rightarrow \sim q) \rightarrow (p \lor r)$
   d. $(p \rightarrow (q \lor r)) \land (p \rightarrow \sim q) \equiv (p \lor r)$

24. Which of the following is false?
   a. $35 \text{ div } 8 = 4$
   b. $219 \text{ mod } 3 = 73$
   c. $17 \text{ div } 2 = 8$
   d. $39 \text{ mod } 7 = 4$

25. Compute the binary sum: $101101 + 1101$
   a. 111001
   b. 101010
   c. 110010
   d. 111010

26. What would be the inductive step using the first principle of mathematical induction to prove that $2^n > n$ for all $n \geq 0$?
   a. to show that $2^0 > 0$
   b. to show that $2^k > k$, where $k$ is an integer and $k \geq 0$, implies that $2^{k+1} > k + 1$ is true
   c. to show that $2^{k+1} > k$ for all $k \geq 0$
   d. to show that $2^k > k$ for all $k \geq 0$

27. Let $A = \{1,2,3,4\}$ and $B = \{x,y,z\}$. If relation $R = \{(1,y), (2,x), (3,z), (4,y)\}$, then which of the following is NOT correct?
   a. $2 \, R \, x$
   b. $(4, x) \notin R$
   c. $R(y) = 4$
   d. Both b and c.
28. Let $A$ be the set of all real numbers and relation $R$ defined on $A$ by
\[ R = \{(x, y) \in A \times A | x^2 + y^2 = 9\}. \] Which of the following is true?

a. Domain of $R$, $D(R) = \{t | 0 \leq t \leq 3\}$
b. Range of $R$, $Im(R) = \{t | 0 \leq t \leq 3\}$
c. Domain of $R$, $D(R) = \text{Range of } R$, $Im(R) = \{t | -3 \leq t \leq 3\}$
d. Both a and b

29. Let $A = \{1,2,3,4,5\}$ and let $R$ be the relation on $A$ defined by
\[ R = \{(1,3), (2,1), (2,2), (2,5), (3,4), (4,3), (4,4), (5,1), (5,3)\}. \] Which of the following is true?

a. $(2,4) \in R^2$
b. $R^2 = R^3$
c. $R^3 = R^4$
d. $R^1 = R^2$

30. Which of the following relations defined on the given set is not a partial order relation?

a. Power set of $A = \{a, b, c, d\}$ under inclusion.
b. $A = \{a, b, c, d\}$ $R = \{(a, a), (b, c), (c, d), (d, d)\}$
c. $A = \{1,2,3,4,5\}$ $R = \{(x,y) | x \geq y\}$
d. $A = \{1,2,3,4,5,6,7,8,9,10\}$ $R = \{(x,y) | x \geq y\}$

31. Which of the following relations on set $A = \{1,2,3,4\}$ is an equivalence relation?

a. $R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$.
b. $R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$.
c. $R = \{(1,1), (2,2), (3,3), (4,4), (1,2)\}$.
d. $R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (2,3), (3,4)\}$.

32. Which of the following relations $f$ are functions from the set $X$ to the set $Y$?

a. $X = \{-3-2, -1, 0, 1, 2\}$, $Y = \{3,4,5,6,7\}$, and $f = \{(-2,3), (-1,6), (0,4), (1,5), (2,7)\}$
b. $X = \{-3-2, -1, 0, 1, 2\}$, $Y = \{3,4,5,6,7\}$, and $f = \{(-3,3), (-2,3), (-1,6), (0,4), (-2,6), (1,5), (2,7)\}$
c. $X = \mathbb{Q} = Y$, defined by $f\left(\frac{n}{m}\right) = n + m$ for all $\frac{n}{m} \in \mathbb{Q}$.
d. $X = \{-3-2, -1, 0, 1, 2\}$, $Y = \{3,4,5,6,7\}$, and $f = \{(-2,3), (0,4), (-3,6), (-1,7), (1,5), (2,7)\}$
33. Let \( A = \{1, 2, 3, 4\} \) and \( B = \{a, b, c, d\} \) be sets. Which of the following arrow diagrams is NOT a function from \( A \) into \( B \)?

34. Which of the following functions does NOT have an inverse?
   a. \( X = \{x \in \mathbb{Z} | -2 < x \leq 5\}, \ Y = \{y \in \mathbb{Z} | 1 < y \leq 8\}, \ f: X \rightarrow Y \) defined by \( f(x) = x + 3 \) for all \( x \in X \).
   b. \( f: \mathbb{Z} \rightarrow \mathbb{Z}, \ f(x) = 4x + 7 \) for all \( x \in \mathbb{Z} \).
   c. \( f: \mathbb{R} \rightarrow \mathbb{R}, \ f(x) = 2x - 5 \) for all \( x \in \mathbb{R} \).
   d. \( f: \mathbb{Q} \rightarrow \mathbb{Q}, \ f(x) = 8x \) for all \( x \in \mathbb{Q} \).

35. Consider the sequence: \( 2, 3, 5, 8, 12, 17, \ldots \). Which of the following is a correct representation this sequence?
   a. \( a_1 = 2 \) and \( a_n = a_{n-1} + 1 + \frac{1}{2}(a_{n-1}) \)
   b. \( a_n = a_{n-1} + a_{n-2} \)
   c. \( a_1 = 2 \) and \( a_n = a_{n-1} + (n - 1) \)
   d. \( a_1 = 2 \) and \( a_n = a_{n-1} + n \)
36. Consider the sum $\sum_{i=0}^{5} i(i - 1)$. If we change the index variable to $j = i - 1$, then the equivalent sum is:
   a. $\sum_{j=-1}^{6} j(j - 1)$
   b. $\sum_{j=-1}^{4} j(j - 1)$
   c. $\sum_{j=-1}^{5} j(j + 1)$
   d. $\sum_{j=-1}^{4} j(j + 1)$

37. Let $A$ be the set of lowercase English alphabet. Suppose $s_1 = oicu$ and $s_2 = rmt$. The concatenation of $s_1$ and $s_2$ is:
   a. $oicurmt$
   b. $\emptyset$, the empty set
   c. 7
   d. $orimctu$

38. Let $S = \{x, y, z\}$. Define $*$ on $S$ by the following multiplication table:

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>y</td>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td>z</td>
<td>x</td>
<td>z</td>
<td>x</td>
</tr>
</tbody>
</table>

   Figure 38. Multiplication table for question 38

   a. $x$ is identity of $(S,*)$
   b. $y$ is identity of $(S,*)$
   c. $z$ is identity of $(S,*)$
   d. $(S,*)$ does not have an identity

39. A solution of the congruence $3x = 7 \pmod{13}$ is:
   a. 3
   b. 11
   c. 14
   d. 21

40. What is the remainder when $1! + 2! + \cdots + 100!$ is divided by 12?
   a. 9
   b. 2
   c. 14
   d. 7
41. Which of the following ISBNs is valid?
   a. 0 − 619 − 015919 − 7
   b. 0 − 201 − 55540 − 8
   c. 0 − 201 − 70982 − 1
   d. 3 − 240 − 19102 − X

42. Find \( T(n) \), the timing function for the code fragment in figure 42.

   ```
   for (i=0; i<n; i++) {
       m=n;
       while (m>1) m=m/4;
   }
   ```

   **Figure 42.** Code Fragment for Question 42.

   a. \( T(n) = 3n[\log_4 n] + 3n + 2 \)
   b. \( T(n) = 3n[\log_2 n] + 3n + 2 \)
   c. \( T(n) = 2n[\log_4 n] + 3n + 2 \)
   d. \( T(n) = 2n[\log_2 n] + 3n + 2 \)

43. Which of the following is a recurrence relation?
   a. \( a_n = 3a_{n-1} + a_{n-2} \forall n \geq 2 \)
   b. \( a_n = 2n^2 - 1 \forall n \geq 1 \)
   c. \( a_{n+1} = \frac{(a_n)!}{n!} \forall n \geq 1 \)
   a. Both a and c

44. Give the recurrence relation for the sequence 1, 5, 25, 125, ...
   a. \( a_1 = 1, a_{n+1} = 5^n \)
   b. \( a_1 = 1, a_{n+1} = 5a_n \)
   c. \( a_n = 5^{2n} \) for \( n \geq 1 \)
   d. \( a_1 = 1, a_n = 5a_{n+1} \) for \( n \geq 1 \)

45. Which of the following is a linear homogeneous recurrence relation?
   a. \( a_n = 3a_{n-1} + a_{n-2} \)
   b. \( a_n = -3a_{n-1} + 7 \)
   c. \( a_n = 3a_{n-1} + \frac{a_{n-2}}{2} \)
   d. \( a_n = a_{n-2} \cdot a_{n-1} \)

46. Find the solution to the recurrence relation \( a_n = 4a_{n-1} \) where \( a_0 = 1 \).
   a. The solution is a sequence, 1, 2, 4, 8, ...
   b. The solution is 4
   c. The solution is a sequence, 1, 4, 16, 64, ...
   d. The solution is 1
47. Find the sum of the sequence $1 + 2 + 3 + \cdots + n$.
   a. $n^2 + n$
   b. $\frac{n(n+1)}{2}$
   c. $\frac{n(n-1)}{2}$
   d. $\frac{2n(n-1)}{2}$

48. What do we normally disregard when we count the number of operations in an algorithm?
   a. relational operations
   b. I/O operations
   c. assignment statements
   d. arithmetic operators

49. Find $T(n)$, the timing function for the code fragment in figure 49?

   ```
   sum=0;
   for (a=0; a<n; a++) {
       for (b=0; b<a; b++) {
           sum++;
       }
   }
   ```

   Figure 49. Code Fragment for Question 49.

   a. $T(n) = \frac{3}{2}n^2 + \frac{5}{2}n + 3$
   b. $T(n) = \frac{3}{2}n^2 + \frac{11}{2}n + 3$
   c. $T(n) = 3n^2 + 4n + 3$
   d. $T(n) = \frac{3}{2}n^2 + \frac{5}{2}n + 3$
50. Given that array L is sorted in ascending order, what is the time complexity of member function search given in figure 50?

```
class List {
    int size;    //Maximum number of available entries in the List.
    int count;   //Number of occupied entries in the List.
    int* L;    //Points to a dynamically allocated array of integers used to implement the list

public:
    int search(int key) //Search the list for an entry whose value matches the key
    { int h=count;l=0;
        while (l<h) {
            int m=(l+h)/2;
            if (L[m]=key) return m;
            if (L[m]<key) h=m-1; else l=m+1;
        }
        return -1; //The key could not be found in the list
    }
}
```

Figure 50. Code Fragment for Question 50.

a. $T(n) = O(n^2)$
b. $T(n) = O(n)$
c. $T(n) = O(\log_2 n)$
d. $T(n) = O(n \log_2 n)$