DEFINITION 1
An algorithm is called recursive if it solves a problem by reducing it to an instance of the same problem with smaller input.

EXAMPLE 1
Give a recursive algorithm for computing $n!$, where $n$ is a nonnegative integer.

Solution:
```c
unsigned int factorial(unsigned int n) {return n>0?n*factorial(n-1):1;}
```

EXAMPLE 2
Give a recursive algorithm for computing $a^n$, where $n$ is a nonnegative integer.

Solution:
```c
double power(double b, unsigned int n) {return n>0?b*power(n-1):1;}
```

EXAMPLE 4
Give a recursive algorithm for computing the greatest common divisor of two non negative integers $a$ and $b$ with $a < b$.

Solution:
```c
unsigned int gcd(unsigned int a, unsigned int b){return a==0?b:gcd(b%a,a);} int main()
{
  for (;;)
  {
    cout << endl;
    cout << "Enter nonnegative integer argument a. ";
    int a;
    cin >> a;
    cout << "Enter nonnegative integer argument b. ";
    int b;
    cin >> b;
    if (a<0||b<0) break;
    if (a>=b) {
      cout << endl << a << " must be less than " << b << ".";
      continue;
    }
    cout << "gcd( " << a << "," << b << ")=" << gcd(a,b);
  }
  return 0;
}
```
EXAMPLE 7  Prove that Algorithm 2, which computes powers of real numbers, is correct.  
*Solution:* We use mathematical induction on the exponent $n$.  

*Basis Step:* If $n = 0$, the first step of the algorithm tells us that $power(b, 0) = 1$. This is correct because $b^0 = 1$ for every nonzero real number $b$. This completes the basis step.

*Inductive Step:* The inductive hypothesis is the statement that $power(b, k) = b^k$ for all $b \neq 0$ for the nonnegative integer $k$. That is, the inductive hypothesis is the statement that the algorithm correctly computes $b^k$. To complete the inductive step, we show that if the inductive hypothesis is true, then the algorithm correctly computes $b^{k+1}$. Because $k + 1$ is a positive integer, when the algorithm computes $b^{k+1}$, the algorithm sets $power(b, k + 1) = b \cdot power(b, k) = b \cdot b^k = b^{k+1}$. This completes the inductive step.